1. Notice that when we round to the thousandths place, we can increase the number by at most .0005. Similarly, when we round to the nearest hundreth and the nearest tenth, we can increase the number by at most .005 and .05 respectivel. Thus, our number must satisfy $x \geq 7 - .0005 - .005 - .05 = .6445$. Notice that if we round .6445 to the thousandths then hundreths then tenths, we get .645 then .65 then .7. So $\boxed{.6445}$ is the smallest possible number we could have started with.

2. Draw the perpendiculars from P to each side of $ABCD$, hitting AB, BC, CD, DA at points E, F, G, H, respectively. Let $AE = DG = w, BE = CG = x, BF = AH = y, CF = DH = z$. Then $PA^2 + PC^2 = (w^2 + y^2) + (x^2 + z^2)$, and similarly, $PB^2 + PD^2 = (x^2 + y^2) + (w^2 + z^2)$. Thus, $PA^2 + PC^2 = PB^2 + PD^2$, so $PB^2 = 15^2 + 20^2 - 7^2 = 24^2$, and $PB = \boxed{24}$.

3. Let $AE = AF = x$, and $BE = 1 - x$. Notice that $[ABCD] = 1, [AEF] = \frac{1}{2}x^2, [BCE] = \frac{1}{2}(1-x)$. Then $[CDFE] = 1 - \frac{1}{2}x^2 - \frac{1}{2}(1-x) = \frac{1}{2}(1+x-x^2)$. Since $1+x-x^2 = \frac{5}{4} - (x-\frac{1}{2})^2 \leq \frac{5}{4}$, and equality is achievable ($x = \frac{1}{2}$), the maximum possible area is $\frac{1}{2} \cdot \frac{5}{4} = \boxed{\frac{5}{8}}$.

4. Let line BC hit the circle again at point E. By power of a point, $AB^2 = BC \cdot BE$, so we find that $BE = 12$. Now let F be the midpoint of CE. We have $OF \perp CE$. We can find $DF = \frac{3}{2}$, and since $OD = 2$, we use the Pythagorean theorem to find that $OF = \frac{\sqrt{7}}{2}$. Now, we have $OC^2 = OF^2 + FC^2 = \frac{7}{4} + \frac{81}{4} = 22$. Thus, the radius of the circle is $\boxed{\sqrt{2}}$.

5. Place points F, G, H inside the square such that EDC, FCB, GBA, and HAD are congruent. Since $ED = DH$ and $\angle EDH = 90° - 15° - 15° = 60°$, triangle EDH is equilateral. Similarly, triangles FCE, GBF, and GAH are equilateral. Thus, $GHEF$ is a rhombus. Since $\angle DEC = 150°$ and $\angle DEH = \angle CEF = 60°$, we have $\angle HEF = 90°$. Thus, $GHEF$ is a square. Since AHG is equilateral and $GHEF$ is a square, AEH is isoceles with $\angle AHE = 150°$. Thus, $\angle AEH = 15°$. Similarly, $\angle BEF = 15°$, so $\angle AEB = \angle HEF - \angle HEA - \angle FEB = \boxed{60°}$.

6. We have $\cos(\alpha + \beta) + \sin(\alpha - \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta + \sin\alpha\cos\beta - \sin\beta\cos\alpha = 0$. Dividing by $\cos\beta$, we have $\cos\alpha - \sin\alpha\tan\beta + \sin\alpha - \cos\alpha\tan\beta = 0$. Since $\tan\beta = \frac{1}{2000}$, we have $(\cos\alpha + \sin\alpha)(1 - \frac{1}{2000}) = 0$, or $\cos\alpha + \sin\alpha = 0$. This is equivalent to $\cos\alpha = -\sin\alpha$, so $\tan\alpha = \boxed{-1}$.

7. We can have:
five numbers divisible by 3 (1 combination)
three 0 mod 3, one 1 mod 3, one 2 mod 3 ($\binom{5}{3} \cdot 5 \cdot 5 = 250$ combinations)
two 0 mod 3, other three all 1 mod 3 or all 2 mod 3 ($2 \cdot (\binom{5}{2} \cdot \binom{5}{3}) = 200$ combinations)
one 0 mod 3, two 1 mod 3, two 2 mod 3 ($5 \cdot 10 \cdot 10 = 500$ combinations)
four 1 mod 3, one 2 mod 3 ($5 \cdot 5 = 25$ combinations)
four 2 mod 3, one 1 mod 3 ($5 \cdot 5 = 25$ combinations)
Adding these up, we find 1001 total combinations possible out of $\binom{15}{5} = 3003$. Thus, the probability is $\frac{1001}{3003} = \boxed{\frac{1}{3}}$.

8. Besides the $n = 1$ case, we can separate the divisors of n into pairs of divisors that have product n. There must be only two of these pairs, so the number n must have 4 divisors.

This is possible only when n is a product of two distinct primes, or when it is the cube of a prime. We list all the numbers under 100 that satisfy these conditions: 1, 6, 8, 10, 14, 15, 21, 22, 26, 27, 33, 34, 35, 38, 39, 46, 51, 55, 57, 58, 62, 65, 69, 74, 77, 82, 85, 86, 87, 91, 93, 94, 95. There are $\boxed{33}$ of these.

9. We suppose that no segments concur at points strictly within the polygon to maximze intersection points.

   Consider any four distinct points. We easily see that if we connect the segments between these four points, only one intersection within the polygon will be made. Furthermore, if we had any intersection, we could find the unique pair of segments that generated that point. Thus, there exists a one-to-one correspondence between intersections and groups of four points, and our answer must be $\binom{n}{4}$.

10. Let $y' = \frac{2}{3}y$ so that $\frac{y}{3} = \frac{y'}{2}$. If we transform all 3 points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ by multiplying their y-coordinates by $\frac{2}{3}$, we get a new set of coordinates $(x_1, y'_1), (x_2, y'_2), (x_3, y'_3)$ that has the same centroid (since $y'_1 + y'_2 + y'_3 = \frac{2}{3}y_1 + y_2 + y_3 = 0$). Also, since this is a vertical stretch by $\frac{2}{3}$, the area of this new triangle will have $\frac{2}{3}$ the area of the original triangle.

    This new triangle satisfies the property that its centroid is identical to its circumcenter, or that all 3 perpendicular bisectors and all 3 medians concur at one point. A perpendicular bisector and a median of a side intersect at the midpoint of that side. The point of concurrency obviously cannot be on the midpoint of each side, so we must have that at least two pairs of perpendicular bisectors and medians are identical. But if a perpendicular bisector and median are the same line, then the other two sides have equal lengths; thus, this triangle must be equilateral.

    We can find the equilateral triangle to have area $3\sqrt{3}$, so multiplying this by $\frac{3}{2}$, we get an area of $\frac{9\sqrt{3}}{2}$.