1. Let α denote the degree measure of each of the angles of $ABCDEFG$. Since ABP is equilateral, $BP = BA = BC$, so BCP is isoceles and $\angle BCP = \angle CPB$. We have $\angle CBP = \alpha - 60^\circ$, so $\angle CPB = \angle BCP = 120^\circ - \frac{\alpha}{2}$. Then $\angle PCD = \alpha - \angle BCP = \frac{3\alpha}{2} - 120^\circ$. By symmetry, $\angle DEP = \angle FEP$, so $\angle DEP = \frac{\alpha}{2}$. Finally, since $\angle CDE = \alpha$, we have $\angle EPC = 360^\circ - \frac{\alpha}{2} - \alpha - (\frac{3\alpha}{2} - 120^\circ) = 480^\circ - 3\alpha$. Computing $\alpha = 180^\circ - \frac{360^\circ}{7} = \frac{900^\circ}{7}$, we find $\angle EPC = \frac{660^\circ}{7}$, so the answer is $\boxed{667}$.

2. Let the dimensions of the prism be $2, x, x + \sqrt{2005}$. By the distance formula, $47^2 = 2^2 + x^2 + (x + \sqrt{2005})^2 = 2009 + 2x(x + \sqrt{2005})$. This gives us $2x(x + \sqrt{2005}) = 47^2 - 2009 = \boxed{200}$.

3. Since $ABCD$ has an incircle, $AD + BC = AB + CD = 5$. Now let $BP = 3x, DP = 8x$. Since $\triangle ABP \sim \triangle DCP$, we have $\frac{AB}{DC} = \frac{AP}{DP} = \frac{BP}{CP}$, so $\frac{AP}{DP} = \frac{BP}{CP} = \frac{1}{4}$. From this, we can find $AP = 2x, CP = 12x$. Now, since $\triangle ADP \sim \triangle BCP$, we have $\frac{AD}{BC} = \frac{AP}{BP} = \frac{2x}{3x}$. Since $AD + BC = 5$, it follows that $AD = 2, BC = 3$. Now, the area of $ABCD$ can be obtained via Brahmagupta's formula: $s = \frac{1+2+3+4}{2} = 5$, $K = \sqrt{(s-a)(s-b)(s-c)(s-d)} = \sqrt{24}$, and $K = rs = 5r$, where r is the inradius of $ABCD$. Thus, $r = \frac{\sqrt{24}}{5}$, so the area of the incircle is $\frac{24\pi}{25}$, and our answer is $\boxed{49}$.

4. Let the two numbers chosen be x, y. If $x < y$, then we must have $y - x > \frac{x+y}{2} \Rightarrow 2y - 2x > x + y \Rightarrow y > 3x$. Similarly, if $x > y$, then we must have $x > 3y$. We can represent this graphically, with the square having opposite corners $(0,0), (1,1)$ representing the possible choices of (x, y). We draw the lines $y > 3x$ and $x > 3y$, and count the shaded area they have, which is two triangles of area $\frac{1}{6}$, giving an area of $\frac{1}{3}$ total. Since the area of the square is 1, it follows that the probability is $\boxed{\dfrac{1}{3}}$.

5. A pentagon has an interior angle of $180^\circ - \frac{360^\circ}{5} = 108^\circ$. The two pentagons and the regular polygon together share an angle of 360°, so the polygon has interior angle $360^\circ - 2 \cdot 108^\circ = 144^\circ$. If the polygon has n sides, then $144^\circ = 180^\circ - \frac{360^\circ}{n}$. Solving for n, we find $n = \boxed{10}$.

6. It is quite helpful to notice that $10040 = 5 \cdot 2008, 6024 = 3 \cdot 2008, 8032 = 4 \cdot 2008$, so this triangle is similar to a $3-4-5$ triangle. The $3-4-5$ triangle has area 6 and semiperimeter 6, so its inradius is 1. Thus, our triangle has 2008 times this inradius, so its incircle has area $\boxed{2008^2\pi}$ or $\boxed{4032064\pi}$.

7. Let $AE = EP = AP = 1$. Then $PT = 1$. Since $AP = TP = PE$, we have that ATE is a right triangle with $\angle EAT = 90^\circ$. Also, we have $\angle AEP = 60^\circ$ since it is part of an equilateral triangle, so this triangle is also a $30-60-90$ triangle. Thus, $AT = \sqrt{3}$. Since $TX = 2$ and $\angle PTA = 30^\circ, \angle XTA = 150^\circ$, we can use the law of cosines to find AX: $AX^2 = 3 + 4 - 4\sqrt{3}\cos 150^\circ = 7 + 4\sqrt{3} \cdot \frac{\sqrt{3}}{2} = 13$. Now, using the law of cosines to find $\cos\angle XAE = \frac{AX^2 + AE^2 - EX^2}{2 \cdot AX \cdot AE} = \frac{-2}{2\sqrt{13}} = \boxed{-\dfrac{\sqrt{13}}{13}}$.

8. Call the three colors red, blue, and green. Say we painted a face red. Then pick a face adjacent to this face. We have a $\frac{2}{3}$ probability of picking a good color (blue or green). After we color this face, we find that the other faces adjacent to the red face have a fixed color

they must be, so we will color them correctly with probability $\frac{1}{3^3}$. Finally, the face opposite the red face must also be red, and we will color this correctly with probability $\frac{1}{3}$. Thus, the probability is $\boxed{\textbf{(B) } \frac{2}{3^5}}$.

9. We know that $\tan(\tan^{-1} a + \tan^{-1} b) = \frac{a+b}{1-ab}$. Now, applying this formula again, we find that $\tan(\tan^{-1} \frac{a+b}{1-ab} + \tan^{-1} c) = \frac{\frac{a+b}{1-ab}+c}{1-\frac{a+b}{1-ab}\cdot c} = \frac{a+b+c(1-ab)}{(1-ab)-(ac+bc)} = \frac{a+b+c-abc}{1-(ab+bc+ca)}$. By Vieta's, $a+b+c = -9, ab+bc+ca = 8, abc = -2$, so our answer is $\frac{-9+2}{1-8} = \boxed{\textbf{(D) } 1}$.

10. Notice that if $f(x) = n$, then $x = n-3$ or $x = 2n$, but the first case is only possible if $n-3$ is odd, or if n is even. Since 27 is odd, we must have $f(f(k)) = 2 \cdot 27 = 54$. Notice that for k to be odd, $f(k)$ must be even. Thus, we cannot have $f(k) = 54 - 3 = 51$, which is odd, and we must have $f(k) = 2 \cdot 54 = 108$. Finally, since k is odd, we have $k = 108 - 3 = 105$. Thus, the sum of the digits of k is $\boxed{\textbf{(B) } 6}$.