1. If the two bases are $1, 5$, then the trapezoid is isoceles. Drawing the perpendiculars to the bases, we see that the height is $\sqrt{4^2 - 2^2} = \sqrt{12} = 2\sqrt{3}$. The area of this trapezoid is then $2\sqrt{3}(1+5)/2 = \boxed{6\sqrt{3}}$. Let us look for other possible configurations of the trapezoid. We cannot have both bases be 4, or the trapezoid would be a parallelogram, which is impossible since the other two lengths are unequal. Thus, exactly one of the bases is 4.

   If the other base is 5, then draw the perpendiculars between the two bases. This cuts the segment 5 into lengths $x, 4, 1-x$. Let the height be h. Then $4^2 - x^2 = h^2, 1^2 - (1-x)^2 = h^2$. Solving for x gives $x = 8$, which is a contradiction.

   If the other base is 1, then draw the perpendiculars between the two bases, cutting the segment 4 into lengths $x, 4, 3-x$. Let the height be h. Then $4^2 - x^2 = h^2 = 5^2 - (3-x)^2$, and solving for x gives $x = 0$. Thus we have another possible trapezoid with a leg perpendicular to the two bases. This trapezoid has height 4, and the sum of the bases is 5, so the area is $\boxed{10}$.

2.

3. Draw a perpendicular from M and D to line AB, hitting the line at points E, X, respectively. We have $DX = 8, AX = 2$. Since ME is parallel to DX, we have $ME = 4, AE = 1$. By the Pythagorean theorem, we have $AM = \sqrt{17}$. Now extend MN until it intersects the extension of line AB at point F. We have $\triangle AME \sim \triangle AFM$, so $\frac{AM}{ME} = \frac{AF}{AM}$, or $AF = \frac{AM^2}{AE} = 17$. Then $BF = 17 - 9 = 8$. Now, we have $\triangle FNB \sim \triangle MAE$, so $\frac{NB}{BF} = \frac{AE}{EM}$, or $\frac{NB}{8} = \frac{1}{4} \iff \boxed{NB = 2}$.

4. Let E be the midpoint of CD, so that $AB = AD = CE = DE = x$. Draw segment BE. Since $AB = DE$ and $\angle ABD = \angle EDB$, we have $\triangle ABD \cong \triangle EDB$, so $BE = x$. Since $DE = BE = CE$, $\triangle BCD$ has a right angle at B. Thus, we have $[BCD] = \frac{1}{2} \cdot 4 \cdot 6 = 12$. Then $[BEC] = [BED] = 6$, and since $\triangle ABD \cong \triangle EDB$, the area of the trapezoid is $6 + 6 + 6 = \boxed{18}$.

5. Draw perpendiculars from A and B to line CD, intersecting it at points E, F respectively. Since $\angle C = 45°$, we have that BFC is a 45-45-90 triangle, so $CF = BF = 6$. Since AE and BF are heights, we have $AE = BF = 6$. Since $\angle A = 120°$, we have $\angle D = 60°$, so ADE is a 30-60-90 triangle, which gives us $DE = 2\sqrt{3}$. Then $CD = 6 + 8 + 2\sqrt{3} = 14 + 2\sqrt{3}$, and the area of the trapezoid is $6 \cdot \frac{8+14+2\sqrt{3}}{2} = \boxed{66 + 6\sqrt{3}}$.

6.

7. In any trapezoid $ABCD$ with bases AB, CD and diagonals intersecting at O, we have $\triangle ABO \sim \triangle CDO$, and $[ADO] = [BCO]$. Let $[ADO] = [BCO] = a$. We have $\frac{a}{4} = \frac{25}{a} = \frac{BO}{OD}$, so $a^2 = 4 \cdot 25$, or $a = 10$. Then the area of the trapezoid is $4 + 25 + 2a = \boxed{49}$.

8. Let the trapezoid be $ABCD$ with $AB = 50, CD = 75, AC = 35, BD = 120$. Extend CD past point C by 50 units to point E. We have that $ABEC$ is a parallelogram, so $BE = AC = 35$. Note that $[BEC] = [BAC] = [BAD]$, since $\overline{CD} \parallel \overline{AB}$. Thus, $[BED] = [BEC] + [BCD] = [BAD] + [BCD] = [ABCD]$. Now, $\triangle BDE$ has side lengths $35, 120, 125$. This is a right triangle, so it has area $\frac{1}{2} \cdot 35 \cdot 120 = \boxed{2100}$. (Alternatively, we could have used Heron's formula.)

9. Draw a line parallel to CD from point B, intersecting AD at point F. $BCDF$ is a parallelogram, so $FD = 7, BF = 8$. Thus, we have $AF = 10$. Obviously, ABF is a right triangle (6-8-10), so $\angle ABF = \angle E = \boxed{90^\circ}$.

10. We have that $[WZQP] = [XYQP]$ and $PQ \parallel XY$, $PQ \parallel WZ$. Also, $XY = WZ$, so the altitude of $XYQP$ equals the altitude of $WZQP$ and both are equal to $\frac{19}{2}$. We have $[ABCD] = 19AB$ by the rectangle. Also, we have $[ABCD] = 4\,[PQXY] = 19(XY + 87)$. Thus, $AB = 87 + XY$. On the other hand, $2AB = AX + WD + YB + CZ + 2XY = XY - 19 + XY - 19 + 2XY = 4XY - 38$. So $AB = 2XY - 19$. Solving the system, we get $AB = \boxed{193}$.