1. Let C be the point that you travel to. We have $CA = AB = 1000$. If $\angle CAB > 60°$, then $\angle ABC < 60°$, so then $BC > AC = 1000$, which is not what we want. Similarly, if $\angle CAB \leq 60°$, then $\angle ABC \geq 60°$, so then $BC \leq AC = 1000$, as desired. So the angle we can travel at can be no more than 60 degrees away from B, giving us a $\frac{120}{360} = \boxed{\frac{1}{3}}$ probability.

2. Since $\angle ACB = 90°$, AB must be the diameter of W. Then the median from C to AB passes through the center of the circle O, so $CO = OB$, and $\angle OCB = \angle OBC = 25°$, so $\angle BOC = \angle AOD = 130°$. Then looking at quadrilateral $AODP$, we have $\angle APD = 360° - 90° - 130° - 90° = \boxed{50°}$.

3. If d is a divisor of 24, then so is $\frac{24}{d}$, so we may pair each factor of 24 with another and multiply them to get 24. We have $24 = 2^3 \cdot 3$, so 24 has $(3+1)(1+1) = 8$ divisors, giving us 4 pairs. Our answer is then $\boxed{4}$.

4. Suppose that Hugo has rolled the maximum product possible with the fixed sum of 70. Note that the average of the dice rolls is $\frac{70}{25} = 2.8$, so there must be some rolls ≤ 2, and some rolls ≥ 3.

   Suppose in this maximum set, Hugo has rolled a 1. Consider also a roll Hugo made with value x with $x \geq 3$. These two rolls have product $1 \cdot x = x$. But if we replaced them with $2, x-1$, which have the same sum, they would have product $2(x-1) = 2x - 2$. Since $x > 2$, we have $2x - 2 > x$, which means that this was not the maximum product possible.

   Therefore, none of Hugo's rolls was a 1. Now suppose that in this maximum set, Hugo has rolled a number k with $k \geq 4$. Also consider a roll 2 that Hugo made, which we know must exist. These two rolls have product $2k$. But if we replaced them with $3, k-1$, then these would have product $3(k-1) = 3k - 3$, and since $k > 3$, we have $3k - 3 > 2k$.

   Thus, Hugo must have rolled only 2's and 3's. If he rolled x 2's and $25 - x$ 3's, then $2x + 3(25 - x) = 70$, which gives us $x = \boxed{5}$.

5. Note that the denominator is equal to $(3x+1)^2+1$. Now if we let $y = 3x+1$ and substitute, we are looking for the min and max of $\frac{y}{y^2+1}$. By expanding out $(y-1)^2 \geq 0$, we have $y^2+1 \geq 2y$, so $\frac{1}{2} \geq \frac{y}{y^2+1}$. By expanding out $(y+1)^2 \geq 0$, we have $y^2+1 \geq -2y$, so $-\frac{1}{2} \geq \frac{y}{y^2+1}$. The equality case in both inequalities is achievable for $y = 1, -1$, or $x = 0, -\frac{4}{3}$, so these are indeed the minimum and maximum. Our answer is then $\frac{1}{2} \cdot -\frac{1}{2} = \boxed{-\frac{1}{4}}$.

6. Let $a = 2^i, b = 2^j$. Then we have $\log_a b = \frac{\log b}{\log a} = \frac{\log 2^j}{\log 2^i} = \frac{j \log 2}{i \log 2} = \frac{j}{i}$. Thus, this problem is equivalent to picking two distinct numbers i, j out of the set $\{1, 2, 3, \ldots, 25\}$ such that $\frac{j}{i}$ is an integer. We see that there are $25 \cdot 24 = 600$ choices for the pair (i, j). We now seek to count how many of these pairs are legal. For $i = 1$, we have 24 possible multiples for j $(2, 3, \ldots, 25)$. For $i = 2$, we have 11 possible choices for j $(2, 4, \ldots, 24)$. For $i = 3$, we have 7 possible choices $(6, 9, \ldots, 24)$. For $i = 4$, we have 5 choices $(8, 12, 16, 20, 24)$. For $i = 5$, we have 4 choices $(10, 15, 20, 25)$. For $i = 6$, we have 3 choices $(12, 18, 24)$. For $i = 7$ and $i = 8$, there are 2 choices each $(14, 21$ and $16, 24)$. For each number from 9 through 12, we have 1 possible choice for j, giving 4 more choices, and for any larger i, we cannot pick any

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j. Thus, there are $24+11+7+5+4+3+2+2+4=62$ possible pairs, and the probability is $\frac{62}{600} = \boxed{\textbf{(B)}\ \frac{31}{300}}$.

7. Notice that for $n \geq 2$, we have

$$f^{[n]}(x) = f^{[n-1]}(f(x)) = \begin{cases} f^{[n-1]}(2x), & \text{if } 0 \leq x \leq \frac{1}{2}; \\ f^{[n-1]}(2-2x), & \text{if } \frac{1}{2} < x \leq 1. \end{cases}$$

Let $g(n)$ be the number of values of x in $[0,1]$ for which $f^{[n]}(x) = \frac{1}{2}$. Then $f^{[n]}(x) = \frac{1}{2}$ for $g(n-1)$ values of x in $[0, \frac{1}{2}]$ and $g(n-1)$ values of x in $[\frac{1}{2}, 1]$. Furthermore $f^{[n]}(\frac{1}{2}) = f^{[n-1]}(1) = 0 \neq \frac{1}{2}$ for $n \geq 2$. Hence $g(n) = 2g(n-1)$ for each $n \geq 2$. Because $g(1) = 2$, it follows that $g(2005) = 2^{2005}$.

8. In a regular n-gon, each angle is equal to $\frac{180(n-2)}{n}$. Thus, the problem tells us that $\frac{(58)(180)(r-2)}{r} = \frac{(59)(180)(s-2)}{s}$. Canceling the 180s and cross-multiplying, we have $s(58r-116) = r(59s-118)$, or $rs - 118r + 116s = 0$. Solving for r, we get $r(s-118) = -116s$, or $r = \frac{116s}{118-s}$. r clearly must be positive, so we must have $s \leq 117$. But the value $s = \boxed{117}$ works, so this is our answer.

9. Since 25% of the tagged fish died off or emigrated, only $60 \cdot .75 = 45$ of the tagged fish are left in the lake. Only 60% of the September lake population was in the lake in May, so $70 \cdot .6 = 42$ of the random sample was originally there during May. Since 3 of this random sample is tagged, the ratio of tagged fish to total fish is $\frac{3}{42}$, and since there are 45 tagged fish remaining, we have $\frac{3}{42} = \frac{45}{x}$, where x is the total number of fish. We can find $x = \boxed{630}$.

10. By the distance formula, we see that $PR = 15$, $QR = 20$, and $PQ = 25$, so $\angle PRQ = 90°$. Therefore, we can extend PR to point I by 10 units such that $\triangle PQI$ is isosceles. The slope of line PR is $\frac{5-(-7)}{-8-1} = -\frac{4}{3}$, so to extend 10 units, we can go 8 units down and 6 units right from R to the point $I = (7, -15)$. Note that since PQI is isoceles, angle P's bisector hits IQ at the midpoint of IQ. We calculate this midpoint to have coordinates $(\frac{7-15}{2}, \frac{-15+(-19)}{2}) = (-4, -17)$. This points and P uniquely determine our line, so we plug them into the equation: $a(-8)+2(5) = a(-4)+2(-17) = -c$, or $a = 11$. Then we must have $c = 8a - 10 = 78$, and $a + c = \boxed{89}$.

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