1. Since $2[BMP] = [MPCA]$, we have $[ABC] = [BMP] + [MPCA] = 3[BMP]$. $\triangle BMP \sim \triangle BCA$ because of shared angle B and a right angle in each. We know that the side ratio between the two is the square root of the area ratio, so $\frac{BP}{BA} = \frac{1}{\sqrt{3}}$. Since $BP = 2$, this means that $BA = 2\sqrt{3}$. Now, $AC^2 = BC^2 - AB^2 = 25 - 12 = 13$, so $AC = \sqrt{13}$. Then the area of ABC is simply $\frac{1}{2}AB \cdot AC = \boxed{\sqrt{39}}$.

2. By the Pythagorean theorem, we have $BE^2 = AB^2 + AE^2 = 1 + \frac{1}{4} = \frac{5}{4}$, so $BE = \frac{\sqrt{5}}{2}$. Also, note that $[BEC] = \frac{1}{2}[ABC] = \frac{1}{4}$. We now attempt to find $[BEF]$ so that we may find $[CEF] = [BEC] - [BEF] = \frac{1}{4} - [BEF]$. We present two methods:

   Solution 1: Let $\angle ABE = \alpha, \angle EBC = \beta$. Note that $\tan\alpha = \frac{AE}{AB} = \frac{1}{2}$, and $\tan(\alpha+\beta) = \tan 45^\circ = 1$. By the tangent addition formula, we have $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{\frac{1}{2} + \tan\beta}{1 - \frac{1}{2}\tan\beta}$. The numerator and denominators are equal, so $\frac{1}{2} + \tan\beta = 1 - \frac{1}{2}\tan\beta \iff \frac{3}{2}\tan\beta = \frac{1}{2} \iff \tan\beta = \frac{1}{3}$. Then $\tan\beta = \frac{EF}{BE}$, so $EF = BE\tan\beta = \frac{\sqrt{5}}{2} \cdot \frac{1}{3} = \frac{\sqrt{5}}{6}$. Then $[BEF] = \frac{1}{2}BE \cdot BF = \frac{1}{2} \cdot \frac{\sqrt{5}}{2} \cdot \frac{\sqrt{5}}{6} = \frac{5}{24}$.

   Solution 2: Let $EF = b$. We have $\frac{[BEF]}{[BEC]} = \frac{BF}{BC}$. We have $[BEF] = \frac{1}{2} \cdot \frac{\sqrt{5}}{2}b$, $[BEC] = \frac{1}{4}$, $BF = \sqrt{\frac{5}{4} + b^2}$, and $BC = \sqrt{2}$. Plugging these all in, we have $\frac{\frac{1}{2} \cdot \frac{\sqrt{5}}{2}b}{\frac{1}{4}} = \frac{\sqrt{\frac{5}{4}+b^2}}{\sqrt{2}}$. Simplifying the left side gives $\sqrt{5}b = \frac{\sqrt{\frac{5}{4}+b^2}}{\sqrt{2}}$, or $\sqrt{10}b = \sqrt{\frac{5}{4} + b^2}$. Squaring this, we have $10b^2 = \frac{5}{4} + b^2$, or $9b^2 = \frac{5}{4}$. Taking the square root of both sides, we have $3b = \frac{\sqrt{5}}{2} \iff b = \frac{\sqrt{5}}{6}$. We then find $[BEF] = \frac{5}{24}$ as in Solution 1. Our answer is then $\frac{1}{4} - \frac{5}{24} = \boxed{\frac{1}{24}}$.

3. Draw the angle bisector of $\angle CAD$, intersecting side BC at point E. Because AE is an angle bisector, we have $CE = CD \cdot \frac{AC}{AC+AD}$. We have $CD = \sqrt{3^2 - 2^2} = \sqrt{5}$, and $AC = 2, AD = 3$, so $CE = \frac{2\sqrt{5}}{5}$. We then have $\tan\alpha = \frac{CE}{AC} = \frac{\frac{2\sqrt{5}}{5}}{2} = \frac{\sqrt{5}}{5}$. Also, $\tan 2\alpha = \frac{CD}{AC} = \frac{\sqrt{5}}{2}$. We now compute $\tan 3\alpha = \tan(2\alpha + \alpha) = \frac{\tan 2\alpha + \tan\alpha}{1 - \tan 2\alpha\tan\alpha}$. Plugging in our values, we obtain $\tan 3\alpha = \frac{\frac{\sqrt{5}}{2} + \frac{\sqrt{5}}{5}}{1 - \frac{\sqrt{5}}{2} \cdot \frac{\sqrt{5}}{5}} = \frac{\frac{7\sqrt{5}}{10}}{1 - \frac{1}{2}} = \frac{7\sqrt{5}}{5}$. Since $\tan 3\alpha = \frac{BC}{AC}$, we have $BC = 2\tan 3\alpha = \frac{14\sqrt{5}}{5}$. Then $BD = BC - CD = \frac{14\sqrt{5}}{5} - \sqrt{5} = \frac{9}{5}\sqrt{5}$. We now find $\frac{CD}{BD} = \frac{\sqrt{5}}{\frac{9}{5}\sqrt{5}} = \boxed{\frac{5}{9}}$.

4. Since a median is cut in a $2:1$ by the centroid, we can let $BG = 2x, GN = x, CG = 2y, GM = y$. Then $MB^2 = BG^2 + GM^2$, or $13 = 4x^2 + y^2$. Similarly, $NC^2 = NG^2 + GC^2$, or $\frac{73}{4} = x^2 + 4y^2$. Adding these together, we have $5(x^2+y^2) = 13 + \frac{73}{4} = \frac{125}{4}$, or $x^2 + y^2 = \frac{25}{4}$. We have $MN^2 = NG^2 + GM^2 = x^2 + y^2 = \frac{25}{4}$, so $MN = \frac{5}{2}$. We have $\triangle AMN \sim \triangle ABC$ by a factor of $1:2$, so $BC = 2MN = \boxed{5}$.

5. The answer cannot be uniquely determined, but we show that $\frac{[ADF]}{[AED]} = \frac{2AF}{AE}$. We have $[ADE] = [ADB]\frac{AE}{AB}, [ADF] = [ADC]\frac{AF}{AC}$, so $\frac{[ADF]}{[AED]} = \frac{[ADC]\frac{AF}{AC}}{[ADB]\frac{AE}{AB}}$. Since $AB = AC$, this reduces to $\frac{[ADC]}{[ADB]} \cdot \frac{AF}{AE}$. But $\frac{[ADC]}{[ADB]} = \frac{BD}{DC} = 2$, so $\frac{[ADF]}{[AED]} = \frac{2AF}{AE}$ as desired.

6. Without loss of generality, let $AC = 1, AB = k$. Since $\triangle ACD \sim \triangle BAD$, we have $\frac{AC}{BA} = \frac{CD}{AD} = \frac{AD}{BD}$. Since $\frac{AC}{BA} = \frac{1}{k}$, we have $\frac{CD}{BD} = \frac{CD}{AD} \cdot \frac{AD}{BD} = \frac{1}{k^2}$.
   With this ratio, we can now begin area ratio chasing. Let $[AEP] = a, [AFP] = b, [BFP] = c, [BDP] = d, [CDP] = e$, and $[CEP] = f$. Because $AP = PD$, we have $b + c = d$ and $e = a + f$. The ratio $\frac{CE}{EA}$ is equal to $\frac{d+e}{b+c}$. Substituting for d, e, we have that this fraction is equal to $\frac{b+c+a+f}{b+c} = 1 + \frac{a+f}{b+c}$. But the ratio $\frac{a+f}{b+c}$ is simply equal to $\frac{1}{k^2}$ because of the base ratio $CD : DE = 1 : k^2$. Then we have $\frac{CE}{EA} = 1 + \frac{1}{k^2}$, so $\frac{CE}{EA} = \frac{k^2+1}{k^2}$. Taking the reciprocal, we have $\boxed{\frac{AE}{EC} = \frac{k^2}{k^2+1}}$.

7. Without loss of generality, we may assume that $BC = 1$. (Dilating the whole figure by some factor will not affect the ratio $\frac{AB-AC}{CD}$.) Since ABC is a 30-60-90 triangle, we have $AC = \sqrt{3}, AB = 2$. By the Angle Bisector Theorem, we have $\frac{CD}{CB} = \frac{AC}{AC+AB}$, so $\frac{CD}{1} = \frac{\sqrt{3}}{\sqrt{3}+2}$. Our answer is then $\frac{AB-AC}{CD} = \frac{2-\sqrt{3}}{\frac{\sqrt{3}}{\sqrt{3}+2}} = \frac{(2-\sqrt{3})(2+\sqrt{3})}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$.

8. We have $\tan < ABC = \frac{AC}{BC}$, or $\tan 15^\circ = \frac{AC}{1}$. Our answer is then $\boxed{\tan 15^\circ = 2 - \sqrt{3}}$.

9. Since $BG = \frac{2}{3}BD$, we have $BG = \frac{16}{3}$. Similarly, $GC = \frac{2}{3}EC = 8$. Then $[BGC] = \frac{1}{2}BG \cdot GC = \frac{64}{3}$, and $[ABC] = 3[BGC] = \boxed{64}$.

10. Let us denote $AC = \sqrt[3]{2} = x$, with $x^3 = 2$. Draw the line parallel to $\overline{BC}$ coming from point D, and extend $\overline{AB}$ so that $\overline{AB}$ and $\overline{BC}$ meet at point E. Since $\triangle ABC \sim \triangle AED$, we have $\frac{AB}{BE} = \frac{AC}{CD}$, or $\frac{1}{BE} = \frac{x}{1}$, or $BE = \frac{1}{x}$. Also, we have $\frac{BC}{ED} = \frac{AC}{AD}$, so $\frac{\sqrt{x^2-1}}{ED} = \frac{x}{x+1}$, or $ED = \frac{(x+1)\sqrt{x^2-1}}{x}$. Now, we have $\tan\angle CBD = \tan\angle BDE = \frac{BE}{ED} = \frac{\frac{1}{x}}{\frac{(x+1)\sqrt{x^2-1}}{x}}$. This simplifies to $\frac{1}{(x+1)\sqrt{x^2-1}} = \frac{1}{\sqrt{(x^2-1)(x+1)^2}} = \frac{1}{\sqrt{x^4+2x^3-2x-1}}$. But since $x^3 = 2$, we have that this is equal to $\frac{1}{\sqrt{2x+4-2x-1}} = \frac{1}{\sqrt{3}}$. Now we know that $\tan\theta = \frac{1}{\sqrt{3}}$, and that θ is acute; thus, we must have $\boxed{\theta = 30^\circ}$.