

- Let triangle ABC have medians AD, BE, CF meeting at centroid G . Show that $\frac{AG}{GD} = \frac{BG}{GE} = \frac{CG}{GF} = 2$.
- Label points in triangle ABC as above. Show that $[AFG] = [BFG] = [BDD] = [CDG] = [CEG] = [AEG]$. What does this say about the comparative areas $[AGB], [BGC], [CGA]$?
- Let $ABCD$ be a convex quadrilateral. Let P be an arbitrary point on side AB . Draw a line through A that is parallel to PD . Likewise, draw a line through B parallel to PC . Let Q be the intersection of these two lines. Prove that $[DQC] = [ABCD]$.
- (AMC12 2006A-18) The function f has the property that for each real number x in its domain, $1/x$ is also in its domain and

$$f(x) + f\left(\frac{1}{x}\right) = x.$$

What is the largest set of real numbers that can be in the domain of f ? (A) $\{x|x \neq 0\}$ (B) $\{x|x < 0\}$ (C) $\{x|x > 0\}$
(D) $\{x|x \neq -1 \text{ and } x \neq 0 \text{ and } x \neq 1\}$ (E) $\{-1, 1\}$

- (AMC12 2006A-20) A bug starts at one vertex of a cube and moves along the edges of the cube according to the following rule. At each vertex the bug will choose to travel along one of the three edges emanating from that vertex. Each edge has equal probability of being chosen, and all choices are independent. What is the probability that after seven moves the bug will have visited every vertex exactly once?
(A) $\frac{1}{2187}$ (B) $\frac{1}{729}$ (C) $\frac{2}{243}$ (D) $\frac{1}{81}$ (E) $\frac{5}{243}$
- (HMMT 2004) Find the largest number n such that $(2004)!$ is divisible by $((n)!)!$
- (HMMT 2005) A true-false test has ten questions. If you answer 5 questions "true" and 5 "false," your score is guaranteed to be at least four. How many answer keys are there for which this is true?
- (HMMT 2005) Find the largest positive integer n such that $1 + 2 + 3 + \dots + n^2$ is divisible by $1 + 2 + 3 + \dots + n$.
- (AIME 1990-4) Find the positive solution to

$$\frac{1}{x^2 - 10x - 29} + \frac{1}{x^2 - 10x - 45} - \frac{2}{x^2 - 10x - 69} = 0.$$

- (AIME 1988-5) Let m/n , in lowest terms, be the probability that a randomly chosen positive divisor of 10^{99} is an integer multiple of 10^{88} . Find $m + n$.