1. Let ω be a cubic root of unity not equal to 1. We see that $\omega^5 + \omega + 1 = \omega^2 + \omega + 1 = 0$; thus, the cubic roots of unity not equal to 1 are roots of this polynomial. Then we have that $x^2 + x + 1$ is a factor of $x^5 + x + 1$. Dividing this out by polynomial long division gives us $\boxed{(x^3 - x^2 + 1)(x^2 + x + 1)}$. (We can be sure the cubic does not factorize further since if it did, it would have a linear factor. But it has no integer roots, so it does not factorize further.)

   Performing a similar process on the other polynomial gives $x^4+x^2+1 = \boxed{(x^2 + x + 1)(x^2 - x + 1)}$.

2. During the course of the twelve-hour period from midnight to noon, the hands begin and end lined up, and line up also eleven other times: a bit after 1, then after 2, etc. The interval between successive line-up times is always the same (to see why, consider the point of view of an ant that lives on the hour hand–the velocity of the minute hand relative to the hour hand is always the same), hence these intervals occur every 12/11 hours. So the answer is 1 oclock plus 1/11 hour, or 1:05:27 and 3/11 sec.

3. Let the escalator have n steps, and move at the rate of r steps per second on its own. Since Sonia takes 20 seconds to get to the top while taking 20 steps, we have $n = 20 + 20r$. Also, since she takes 16 seconds to get to the top while taking 32 steps, we have $n = 32 + 16r$. We have $20 + 20r = 32 + 16r$, or $r = 3$, and plugging this back in, we have $n = \boxed{80}$.

4. From the second equation, we have $(a + b)c = 23$. Since 23 is prime, we must either have $a + b = 1$ or $a + b = 23$. But the first is impossible, since a, b are both positive integers. Thus, we must have $a + b = 23, c = 1$. Then from the top equation, we have $ab + b = 44$. Plugging in $a = 23 - b$, we obtain $b(23 - b) + b = 24b - b^2 = 44$, which has solutions $b = 2, 22$. Both of these correspond to possible solutions of a, namely $21, 1$, so we have $\boxed{\textbf{(C)}\ 2}$ possible solutions.

5. Let $OM = MX = a, ON = NY = b$. By the Pythagorean theorem, we have $(2a)^2 + b^2 = XN^2 = 361, a^2 + (2b)^2 = YM^2 = 484$. Adding these together gives $5(a^2 + b^2) = 845$, or $a^2 + b^2 = 169$. Then $MN = 13, XY = \boxed{\textbf{(B)}\ 26}$.

6. Note that the real coefficients will correspond with an even x power, since i must be taken to an even power to be real. We are then looking for the sum of the coefficients of even degree terms. But this is just $\frac{(1+ix)^{2009}+(1-ix)^{2009}}{2}$, since the odd powers of x will cancel out, and the even ones will be added twice. Then the sum of the coefficients of this polynomial is $\frac{(1+i)^{2009}+(1-i)^{2009}}{2}$. Since $(1+i)^4 = (1-i)^4 = -4$, this is equal to $\frac{(-4)^{502}(1+i+1-i)}{2} = 4^{502} = 2^{1004}$. The log base 2 of this is $\boxed{1004}$.

7. Note that $2002 = 11 \cdot 13 \cdot 14$. Thus, there are 11 multiples of $13 \cdot 14$ less than or equal to 2002. Of course, one of these is 2002, so we only count 10 multiples of $13 \cdot 14$. Similarly, we count 12 multiples of $11 \cdot 14$, and 13 multiples of $11 \cdot 13$. Then our answer is $11 \cdot 10 + 13 \cdot 12 + 14 \cdot 13 = \boxed{\textbf{(A)}\ 448}$.

8. We can pick a set (unordered) of four distinct numbers from the set $\{1, 2, 3, \dots, 9\}$ in $\binom{9}{4} = 126$ ways. Then given these four numbers, we can arrange them in 5 ways to meet the conditions. For example, for the set $\{1, 2, 3, 4\}$, we see that since $c > b, d$, we must have $c = 3, 4$. If $c = 3$,

then either $d = 2, b = 1, a = 4$, or $d = 1, b = 2, a = 4$. If $c = 4$, then we are restricted only to $b < a$, which we can do in 3 ways.

Our answer is then $126 \cdot 5 = \boxed{630}$.

9. Since n is divisible by 15, its last digit must be 0 or 5. Since only 0 is available, its last digit is 0. Now, n must have at least three 8s to be a multiple of 3, so the smallest possible value is 8880. Dividing this by 15, we get $\boxed{592}$.

10. Since $a^5 = b^4$, a must be a perfect fourth power. Similarly, c must be a perfect square. Let $a = m^4, c = n^2$. Then $c - a = n^2 - m^4 = (n - m^2)(n + m^2) = 19$. Since 19 is prime, the factors $n - m^2, n + m^2$ must be equal to 1 and 19 respectively. (The other order is impossible since $n - m^2 < n + m^2$.) Solving the system $n - m^2 = 1, n + m^2 = 19$, we have $n = 10, m^2 = 9$. Plugging this back into our original equations, we have $b^4 = a^5 = (m^4)^5 \Rightarrow b = m^5 = 243$, and $d^2 = c^3 = (n^2)^3 \Rightarrow d = n^3 = 1000$, and our answer is $1000 - 243 = \boxed{757}$.