1. Translate the points so that A is on the origin and B is on $(99, 999)$. We have that the line between the two points is $y = \frac{999x}{99} = \frac{111x}{11}$. Thus, x can be any multiple of 11 not equal to 0 or 99; there are 8 of these: $11, 22, 33, \ldots, 77, 88$, giving an answer of $\boxed{\textbf{(D)}\ 8}$.

2. If a dot is removed from an side with an odd amount of dots, the probability of rolling an odd is $\frac{1}{3}$, and if a dot is removed from a side with an even amount of dots, the probability of rolling an odd is $\frac{2}{3}$. Also, the probability of picking a dot from the side with i dots is $\frac{i}{1+2+3+4+5+6} = \frac{i}{21}$. Thus, our desired probability is $\frac{1}{21}\cdot\frac{1}{3} + \frac{2}{21}\cdot\frac{2}{3} + \frac{3}{21}\cdot\frac{1}{3} + \frac{4}{21}\cdot\frac{2}{3} + \frac{5}{21}\cdot\frac{1}{3} + \frac{6}{21}\cdot\frac{2}{3} = \frac{1+4+3+8+5+12}{63} = \boxed{\textbf{(D)}\ \frac{11}{21}}$.

3. Taking $\log_b \sin x = a$ to the b-th power of both sides, we have $\sin x = b^a$. Now, we have $\log_b \cos x = \log_b \sqrt{1 - \sin^2 x} = \frac{1}{2}\log_b(1 - \sin^2 x)$. Substituting in for $\sin x$, we have $\log_b \cos x = \boxed{\textbf{(D)}\ \frac{1}{2}\log_b(1 - b^{2a})}$.

4. We have $f(2^n) = 2^{n-1} f(2^{n-1}) = 2^{n-1}\cdot 2^{n-2} f(2^{n-2})$. Continuing in this pattern, we see that $f(2^n) = 2^{n-1}\cdot 2^{n-2}\cdot 2^{n-3}\cdots 2^1 f(2^0)$. Since $f(2^0) = f(1) = 1$, we have that $f(2^n) = 2^{(n-1)+(n-2)+(n-3)+\ldots+1} = 2^{\frac{n(n-1)}{2}}$. Plugging in $n = 100$, we ave that $f(2^{100}) = \boxed{\textbf{(D)}\ 2^{4950}}$.

5. We have that $\angle ADC = \angle AEC = 90^\circ$, so it follows that $ADEC$ is cyclic. Let $EC = x$. By Power of a Point, we have that $BD \cdot BA = BE \cdot BC$, or $3 \cdot 8 = 2(2 + x) \iff 12 = 2 + X$. Thus, our answer is $\boxed{10}$.

6. Note that $\log_w x = \frac{\log x}{\log w} = \frac{1}{\log_x w} = \frac{1}{24}$. Similarly, $\log_w y = \frac{1}{40}, \log_w xyz = \frac{1}{12}$. It then follows that $\log_w z = \log_w xyz - \log_w x - \log_w y = \frac{1}{12} - \frac{1}{24} - \frac{1}{40} = \frac{1}{60}$. We then have $\log_z w = \frac{1}{\log_w z} = \boxed{60}$.

7. Let $\sqrt{x^2 + 18x + 45} = y$, with $y \geq 0$. Substituting in, we have $y^2 - 15 = 2y \iff y^2 - 2y - 15 = (y-5)(y+3) = 0$. Since y is nonnegative, it follows that the only solution is $y = 5$. Substituting back in, we have $\sqrt{x^2 + 18x + 45} = 5$, or $x^2 + 18x + 45 = 25 \iff x^2 + 18x + 20$. The discriminant, $18^2 - 4 \cdot 20$, is clearly positive, so both roots of this quadratic are real. Thus, our answer, by Vieta's, is $\boxed{20}$.

8. Note that $\angle XMN = \angle XNM = 45^\circ$, so $\angle MXN = 90^\circ$. Since $\angle MON = 90^\circ$, it follows that $MONX$ is cyclic. We then have $\angle MOX = \angle MNX = 45^\circ$, so it follows that X must lie on the line $y = x$ in the first quadrant. We can clearly reach any point (a, a) $(a > 0)$ by picking the points $M = (0, a), N = (a, 0)$, so all such points are possible. Thus, our answer is all points on the line $y = x$ in the first quadrant.

9. Note that $\angle BDP = \angle BFP = 90$; thus, $BFPD$ is cyclic. (Similarly, $FECP$ is cyclic. Also, $ABPC$ is cyclic by the problem statement.) Since $BFPD$ is cyclic, we have that

$$\angle BFD = \angle BPD = 90 - \angle PBD.$$

Also, since $ABPC$ is cyclic, we have

$$\angle PBD = 180 - \angle ABP = \angle ACP = \angle ECP.$$

Since $FECP$ is cyclic, we have

$$\angle ECP = 90 - \angle EPC = 90 - \angle EFC.$$

Now, plugging in backwards, we have $\angle PBD = 90 - \angle EFC$, so $\angle BFD = 90 - (90 - \angle EFC) = \angle EFC$. This implies that D, F, E are collinear by vertical angles.

10. Let P be the intersection point of the circumcircles of BZX, CXY. We have that $BZPX$ and $CXPY$ are cyclic. We have $\angle ZPY = 360 - \angle ZPX - \angle XPY = 360 - (180 - \angle B) - (180 - \angle C) = \angle B + \angle C$. Thus, we have $\angle A + \angle ZPY = \angle A + \angle B + \angle C = 180$, or that $AZPY$ is cyclic, as desired.