1. (Let us take all angle measures in degrees). We have $\sin(75) = \sin(30+45) = \sin 30 \cos 45 + \cos 30 \sin 45 = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\dfrac{\sqrt{2}+\sqrt{6}}{4}}$. Similarly, we have $\cos(30+45) = \cos 30 \cos 45 - \sin 30 \sin 45 = \boxed{\dfrac{\sqrt{6}-\sqrt{2}}{4}}$.

2. First, since 372=360+12, we can replace $\sin 372^\circ$ with $\sin 12^\circ$. Our expression then becomes $\sin 18^\circ \cos 12^\circ + \cos 18^\circ \sin 12^\circ$, which is just $sin(18^\circ + 12^\circ) = \sin 30^\circ = \boxed{\dfrac{1}{2}}$.

3. Letting $\alpha + \beta = \gamma$, we have $\sin\gamma \sin\beta + \cos\gamma\cos\beta = \cos(\gamma - \beta)$, which, of course, is simply $\boxed{\cos\alpha}$.

4. We will assume all angle measures are represented in degrees. Let $S = \cos 20 \cos 40 \cos 80$. Then, multiplying by $\sin 20 \sin 40 \sin 80$, we have

$$\begin{aligned}\sin 20 \sin 40 \sin 80 S &= (\sin 20 \cos 20)(\sin 40 \cos 40)(\sin 80 \cos 80)\\ &= (\frac{1}{2}\sin 40)(\frac{1}{2}\sin 80)(\frac{1}{2}\sin 160)\\ &= \frac{1}{8}\sin 40 \sin 80 \sin 20.\end{aligned}$$

Dividing again by $\sin 20 \sin 40 \sin 80$, we have that $S = \boxed{\dfrac{1}{8}}$.

5. Squaring the equation, we have $\sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha = .04$. Substituting $\sin^2\alpha + \cos^2\alpha = 1$, and $2\sin\alpha\cos\alpha = \sin 2\alpha$, we have $\sin 2\alpha = .04 - 1 = \boxed{-.96}$.

6. Squaring the first equation, we have $\sin^2 a + \sin^2 b + 2\sin a \sin b = \frac{5}{3}$, and squaring the second, we have $\cos^2 a + \cos^2 b + 2\cos a\cos b = 1$. Adding the two, we have $\sin^2 a + \cos^2 a + \sin^2 b + \cos^2 b + 2(\cos a \cos b + \sin a \sin b) = 2 + 2\cos(a-b) = \frac{8}{3}$. Simplifying, we have $\cos(a-b) = \boxed{\dfrac{1}{3}}$.

7. Let all angle measurements be in degrees. We have

$$\sin 50(1+\sqrt{3}\tan 10) = \frac{\sin 50}{\cos 10}(\cos 10 + \sqrt{3}\sin 10).$$

Note that $\cos x + \sqrt{3}\sin x = 2\sin(x+30)$, so the factor is equal to $2\sin(10+30) = 2\sin 40$. Then, we have

$$\begin{aligned}\frac{2\sin 50 \sin 40}{\cos 10} &= \frac{2\cos 40 \sin 40}{\cos 10}\\ &= \frac{\sin 80}{\cos 10}\\ &= \frac{\cos 10}{\cos 10}\\ &= \boxed{1}.\end{aligned}$$

8. We have $(1+\tan A)(1+\tan B) = \tan A \tan B + \tan A + \tan B + 1$. Recall that $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$. But since $\tan(A+B) = \tan 45° = 1$, we have that $1 - \tan A \tan B = \tan A + \tan B$. Substituting this in, we have $\tan A \tan B + \tan A + \tan B + 1 = \boxed{\textbf{(B) } 2}$.

9. We prove the two following **sum-to-product identities**: $\sin a + \sin b = 2\sin(\frac{a+b}{2})\cos(\frac{a-b}{2})$, and $\cos a + \cos b = 2\cos(\frac{a+b}{2})\cos(\frac{a-b}{2})$. For both of these, let us denote $u = \frac{a+b}{2}, v = \frac{a-b}{2}$, and note that $a = u+v, b = u-v$.

   For the first identity, we have

$$\begin{aligned}\sin a + \sin b &= \sin(u+v) + \sin(u-v)\\ &= \sin u \cos v + \sin v \cos u + \sin u \cos v - \sin v \cos u\\ &= 2\sin u \cos v,\end{aligned}$$

   as desired.

   Similarly, we have

$$\begin{aligned}\cos a + \cos b &= \cos(u+v) + \cos(u-v)\\ &= \cos u \cos v - \sin u \sin v + \cos u \cos v + \sin u \sin v\\ &= 2\cos u \cos v.\end{aligned}$$

   Now, using our identities, we may simplify the expression:

$$\begin{aligned}\frac{\sin 10° + \sin 20°}{\cos 10° + \cos 20°} &= \frac{2\sin 15° \cos 5°}{2\cos 15° \cos 5°}\\ &= \frac{\sin 15°}{\cos 15°}\\ &= \tan 15°\end{aligned}$$

   Thus, our answer is $\boxed{\textbf{(D) } \tan 15°}$.

10. Let a, b be reals with $\cot a = x, \cot b = y$. Note that $\cot(\cot^{-1} x + \cot^{-1} y) = \cot(a+b) = \frac{1}{\tan(a+b)} = \frac{1 - \tan a \tan b}{\tan a + \tan b} = \frac{1 - \frac{1}{xy}}{\frac{1}{x} + \frac{1}{y}}$. Multiplying by xy, we see that this is equal to $\frac{xy-1}{x+y}$.

    Our work now becomes simple: We have $\cot(\cot^{-1} 3 + \cot^{-1} 7) = \frac{3 \cdot 7 - 1}{3+7} = \frac{20}{10} = 2$, so $\cot^{-1} 3 + \cot^{-1} 7 = \cot^{-1} 2$. Similarly, we have $\cot^{-1} 13 + \cot^{-1} 21 = \cot^{-1} \frac{13 \cdot 21 - 1}{13+21} = \cot^{-1} \frac{272}{34} = \cot^{-1} 8$. Our answer is then $10\cot(\cot^{-1} 2 + \cot^{-1} 8) = 10\frac{2 \cdot 8 - 1}{2+8} = \boxed{15}$.