11. Plugging in $x = 180°, y = \beta$ into the formula $\sin(x - y) = \sin x \cos y - \cos x \sin y$, we have $\sin(180° - \beta) = \sin 180° \cos\beta - \cos 180° \sin\beta = \sin\beta$, as desired.

12. Using the formula $[ABC] = \frac{1}{2}bc\sin A$, we have $\frac{[ADE]}{[ABC]} = \dfrac{\frac{1}{2}\cdot 4\cdot 6\sin\angle A}{\frac{1}{2}\cdot 8\cdot 10\sin\angle A} = \frac{24}{80} = \frac{3}{10}$.

13. Let the sides have length $2x, 3x, 4x$. The smallest angle is opposite the smallest side, so we want the angle opposite of the side with length $2x$. Then, by the law of cosines, we have $(2x)^2 = (3x)^2 + (4x)^2 - 2(3x)(4x)\cos\theta \iff 21x^2 = 24x^2\cos\theta \iff \boxed{\dfrac{21}{24}} = \cos\theta$.

14. Let the altitude of C intersect $\overline{AB}$ at D. Then $AD = b\cos A, BD = a\cos B$, so $c = AD + BD = b\cos A + a\cos B$, as desired.

    Remark: This formula still works if ABC has a right or obtuse angle.

15. Let K denote the area of ABC. We claim that $K = 2R^2\sin A\sin B\sin C$.

    Proof. By the extended law of sines, we have $a = 2R\sin A, b = 2R\sin B$. Then,

$$\begin{aligned} K &= \frac{1}{2}ab\sin C \\ &= \frac{1}{2}(2R\sin A)(2R\sin B)\sin C \\ &= 2R^2\sin A\sin B\sin C, \end{aligned}$$

    as desired. □

    Now, plugging in, we have $[ABC] = 2\cdot 5^2\cdot\sin 45°\sin 60°\sin 75° = \boxed{\dfrac{25(3+\sqrt{3})}{4}}$.

16. By the law of cosines on triangle ABD, we have $AB^2 = 2^2 + (2\sqrt{3})^2 - 2\cdot 2\cdot 2\sqrt{3}\cdot\cos 30° = 4$, or $AB = 2$. A similar calculation gives $BC = \sqrt{3}$. Noting that $BC^2 + CD^2 = 3 + 9 = 12 = BD^2$, we see that $\angle BCD = 90°$. Also, since $AD = AB = 2$, we have $\angle ABD = \angle ADB = 30°$, and $\angle BAD = 120°$. From this, we find that $\angle ABC = 90°$. Now, $[ABC] = \frac{1}{2}bh = \frac{1}{2}\cdot 2\cdot\sqrt{3} = \boxed{\sqrt{3}}$.

17. Let O be the center of $ABCD$, and let X be the midpoint of AB. By symmetry, we see that $KLMN$ is a square. We see that $OK = OX + XK = 2 + 4\sin 60° = 2 + 2\sqrt{3}$. Then $KN = \sqrt{2}OK = 2\sqrt{2}(1+\sqrt{3})$, and the area of $KLMN$ is $KN^2 = (2\sqrt{2})^2(1+\sqrt{3})^2 = 8(4 + 2\sqrt{3}) = \boxed{\text{(D) } 32 + 16\sqrt{3}}$.

18. Using the law of sines of triangle OAB, we have $\frac{1}{\sin 30°} = \frac{OB}{\sin\angle OAB}$, or $OB = 2\sin\angle OAB$. Clearly, this is maximized when $\angle OAB = 90°$, so the maximum possible value is $\boxed{\text{(D) } 2}$.

19. Let the median from A intersect $\overline{BC}$ at D. We have $AD = BC = x$. Using the formula for the length of the median (see Problem Set 2.1, Problem 6), we have $2AD = \sqrt{2AB^2 + 2AC^2 - BC^2}$, or $2x = \sqrt{2 + 8 - x^2}$. Squaring gives $4x^2 = 10 - x^2$, or $x^2 = 2$. Thus, $x = BC = \boxed{\text{(C) } \sqrt{2}}$.

20. Using the median formula, we have $AM = \frac{\sqrt{2(5^2+7^2)-6^2}}{2} = \boxed{2\sqrt{7}}$.