1. Since $\sin^2\alpha + \cos^2\alpha = 1$, we have $\cos\alpha = \pm\sqrt{1-\sin^2\alpha}$.
   Then, we have $\cos\alpha = \pm\sqrt{1-(\frac{8}{17})^2} = \boxed{\pm\frac{15}{17}}$, $\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \boxed{\pm\frac{8}{15}}$, and $\cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \boxed{\pm\frac{15}{8}}$.

2. Substituting $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$ and clearing denominators gives $5\sin\alpha = 2\cos\alpha$, or $\frac{5}{2}\sin\alpha = \cos\alpha$. Plugging this in for $\cos\alpha$, we have $\frac{\sin\alpha - 2\cos\alpha}{\cos\alpha - 3\sin\alpha} = \frac{\sin\alpha - 5\sin\alpha}{\frac{5}{2}\sin\alpha - 3\sin\alpha} = \frac{-4\sin\alpha}{-\frac{1}{2}\sin\alpha} = \boxed{8}$.

3. Recalling that $\cos(180° - x) = -\cos x$, we have $\cos C = -\cos(180 - C) = -\cos(A+B) = -\cos A\cos B + \sin A\sin B$. We can calculate $\cos A = \pm\frac{3}{5}, \sin B = \frac{5}{13}$. (We can ignore the negative value for $\sin B$ because $\sin x$ is nonnegative for all angles between 0 and 180°.) We now plug in: $-\cos A\cos B + \sin A\sin B = -(\pm\frac{3}{5})(\frac{12}{13}) + (\frac{4}{5})(\frac{5}{13}) = \frac{\pm 36+20}{65} = \boxed{-\frac{16}{65}, \frac{56}{65}}$.

4. Since $(-5,0)$ and $(5,0)$ are 10 units away, the third vertex must be on the line $y = 2$ or $y = -2$ to form a triangle of area 10. The possible points $(5\cos\theta, 5\sin\theta)$ form a circle with radius 5 centered at the origin. Each line intersects this circle twice, giving a total of 4 points. Thus, the answer is $\boxed{\text{(C) } 4}$.

5. Let the center be O, and let the two points on the circumeference of the circle forming the boundaries of the sector be A, B. We are looking for the circumradius of triangle OAB. We have $\angle AOB = \theta, \angle OAB = \angle OBA = 90 - \frac{\theta}{2}$. By the extended law of sines, we have $2R = \frac{OA}{\sin OBA} = \frac{6}{\sin(90-\frac{\theta}{2})} = \frac{6}{\cos\frac{\theta}{2}} = 6\sec\frac{1}{2}\theta$. Dividing by 2, we see that the answer is $\boxed{\text{(D) } 3\sec\frac{1}{2}\theta}$.

6. Square the equation, obtaining $\sin^2 x + \cos^2 x + 2\sin x\cos x = \frac{1}{25}$, or $\sin x\cos x = -\frac{12}{25}$. By Vieta's, we see that $\sin x, \cos x$ are the roots of the quadratic $t^2 - \frac{1}{5}t - \frac{12}{25}$. This factors as $(t-\frac{4}{5})(t+\frac{3}{5})$, so either $\sin x = \frac{4}{5}, \cos x = -\frac{3}{5}$, or $\sin x = -\frac{3}{5}, \cos x = \frac{4}{5}$. Since $0 \le x < \pi$, the second case is impossible, as $\sin x$ cannot be negative. Thus, our answer is $\frac{\sin x}{\cos x} = \boxed{\text{(A) } -\frac{4}{3}}$.

7. We have $\frac{2ab}{a^2-b^2} = \frac{\sin x}{\cos x}$, so $\sin x = (2ab)k, \cos x = (a^2-b^2)k$, for some value k. Squaring both equations and adding, we have

$$\begin{aligned} 1 &= k^2(4a^2b^2 + (a^4 - 2a^2b^2 + b^4)) \\ &= k^2(a^4 + 2a^2b^2 + b^4) \\ &= k^2(a^2+b^2)^2. \end{aligned}$$

   Thus, we have $k = \frac{1}{a^2+b^2}$, and $\sin x = \boxed{\text{(E) } \frac{2ab}{a^2+b^2}}$.

8. Squaring the equation, we have $\sin^2 x = 9\cos^2 x$. Adding $\cos^2 x$ to both sides, we have $1 = 10\cos^2 x$. Noting that $\sin x \cos x = (3\cos x)\cos x = 3\cos^2 x$, we can multiply $1 = 10\cos^2 x$ by $\frac{3}{10}$ on both sides to find the answer of $\boxed{\text{(E) } \dfrac{3}{10}}$.

9. Applying the law of cosines on triangle AMN, we see that $MN^2 = AN^2 + AM^2 - 2AN \cdot AM \cos\theta$, or $2 = 5 + 5 - 2 \cdot 5\cos\theta \iff \cos\theta = \frac{4}{5}$. Then we have $\sin\theta = \pm\frac{3}{5}$. Since θ is acute, we can discard the negative solution, so our answer is $\boxed{\text{(B) } \dfrac{3}{5}}$.

10. Clearing denominators, we have

$$(\sqrt{3} - 1)\cos x + (\sqrt{3} + 1)\sin x = 4\sqrt{2}\sin x \cos x = 2\sqrt{2}\sin 2x,$$

where we have applied the double angle formula on the RHS. Dividing this by two, we have

$$\begin{aligned}\sqrt{2}\sin 2x &= \left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)\cos x + \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)\sin x \\ &= (\sin 60^\circ - \sin 30^\circ)\cos x + (\cos 30^\circ + \cos 60^\circ)\sin x \\ &= \sin 60^\circ \cos x + \cos 60^\circ \sin x + \cos 30^\circ \sin x - \sin 30^\circ \cos x \\ &= \sin(x + 60^\circ) + \sin(x - 30^\circ)\end{aligned}$$

By the sum-to-product identity, we have $\sin A + \sin B = 2\sin(\frac{A+B}{2})\cos(\frac{A-B}{2})$. Thus, the above is equal to $2\sin(x + 15^\circ)\cos 45^\circ = \sqrt{2}\sin(x + 15^\circ)$. Our equation then reduces to $\sin 2x = \sin(x + 15^\circ)$. This equation is true when $2x = x + 15$, or $2x + (x + 15) = 180$. The two equations give solutions $15, 55$, which correspond to the radian measures of $\boxed{\dfrac{\pi}{12}, \dfrac{11\pi}{36}}$.