



Math Olympiad and Problem Solving Programs
G210 - Introductory Math Olympiad
Problem Set 5.1 - Prime Factorization

Name:

Date:

1. $\boxed{16}$. Prime factorize $2010 = 2 \times 3 \times 5 \times 67$. The number of factors is $2^4 = 16$.
2. $\boxed{12}$. Regroup N into square numbers: $N = 2^3 \cdot 3^4 \cdot 5^3 = (2^2 \cdot 3^4 \cdot 5^2)(2 \cdot 5) = (2 \cdot 3^2 \cdot 5)^2(2 \cdot 5)$. Now, we count the number of factors of $2 \cdot 3^2 \cdot 5$. The answer is $2 \times 3 \times 2 = 12$.
3. $\boxed{312}$. We need to convert n into prime factorization form before applying the factor-counting formula.

$$n = 2^4 \cdot 3^5 \cdot 4^6 \cdot 6^7 = 2^4 \cdot 3^5 \cdot (2^2)^6 \cdot (2 \cdot 3)^7 = 2^{23} \cdot 3^{12}.$$

So, the number of factors is $24 \times 13 = 312$.

4. $\boxed{4}$. Since $12 = 2^2 \cdot 3$, we regroup the number into a multiple of $2^2 \cdot 3$.

$$2^2 \cdot 3^2 \cdot 5 = (3 \cdot 5)(2^2 \cdot 3)$$

Thus, it has 4 factors that are multiples of 12.

5. $\boxed{6}$. For odd factors, we count only the odd prime factors of n , which are $3^1 \cdot 7^2$. Thus, it has $2 \times 3 = 6$ odd factors.
6. $\boxed{6}$. If n has two factors, it must be a prime number. Thus, n^5 has 6 factors.
7. $\boxed{48}$. Since $10 = 1 \times 10$ and 2×5 , it must have only two forms: p^9 and $p \cdot q^4$ where p and q are prime numbers. The least number is $3 \cdot 2^4 = 48$.
8. $\boxed{30}$. Since $8 = 1 \times 8$, 2×4 and $2 \times 2 \times 2$, it must have only three forms: p^7 , $p \cdot q^3$ and $p \cdot q \cdot r$ where p , q , and r are prime numbers. The least number is $2 \cdot 3 \cdot 5 = 30$.
9. $\boxed{12}$. Since $6 = 1 \times 6$ and 2×3 , it must have only two forms: p^5 and $p \cdot q^2$ where p and q are prime numbers. The least number is $3 \cdot 2^2 = 12$.
10. $\boxed{496}$. From 1 to 15, 12 has the largest number of factors. So, $b = 12 = 2^2 \cdot 3$. For largest exponent, we want $n = 15$. Thus,

$$b^n = (2^2 \cdot 3)^{15} = 2^{30}3^{15},$$

which has 496 factors.