



Math Olympiad and Problem Solving Programs
G210 - Introductory Math Olympiad
Problem Set 3.2 - Counting Techniques Solutions

Name:

Date:

1. $\boxed{125}$. There are five odd digits so we have $5^3 = \boxed{125}$.
2. $\boxed{24}$. This is the same as seating five people around a table. We have $\frac{5!}{5} = \boxed{24}$ ways.
3. $\boxed{15}$. For a set of 4 elements, there is a total of $2^4 = 16$ subsets, of which we must remove the empty set. Thus, $16 - 1 = \boxed{15}$.
4. $\boxed{8}$. Use the distance between two dots as the unit length of 1, we have a total of $\boxed{8}$ cases:
 $1 \times 1, 2 \times 2, 3 \times 3, 4 \times 4, \sqrt{2} \times \sqrt{2}, \sqrt{5} \times \sqrt{5}, \sqrt{8} \times \sqrt{8}, \sqrt{10} \times \sqrt{10}$
5. $\boxed{72}$. There are 3 ways to select unit digit, 2 ways to select the thousandth digit, 4 ways to select the hundredth digit, and 3 ways to select the tens digit. Thus, we have $2 \times 4 \times 3 \times 3 = \boxed{72}$.
6. $\boxed{130}$. The first generation has 1 female Amanda. The second generation has 3 female. The third generation has 36 children which gives 18 female. The fourth generation has 216 children which gives 108 female. The total is $1 + 3 + 18 + 108 = \boxed{130}$.
7. $\boxed{420}$. There are $7!$ ways to arrange 7 letters, but we have repeats of N and O. Therefore, we have $\frac{7!}{2!3!} = \boxed{420}$.
8. $\boxed{45}$. There are $5 \times 5 = 25$ handshakes between the players. There are $10 \times 2 = 20$ handshakes between the players and the refs. Total handshakes are $25 + 20 = \boxed{45}$.
9. $\boxed{20}$. Use the R to represent a move to the right and U a move up, the path is a configuration of 6 letters: RRRUUU. Thus, we have $\frac{6!}{3!3!} = \boxed{20}$.
10. $\boxed{15}$. The number of games is $\binom{6}{2} = \frac{6!}{4!2!} = \boxed{15}$.