

- Each edge is adjacent to 2 faces, so we need at least 3 black edges to cover all 6 faces. It is easy to find such an arrangement, so the answer is **(B) 3**.
- From any pair of two distinct letters from A to Y, there is exactly one way to arrange it in alphabetical order. Thus, there are $\binom{25}{2} = \frac{25 \cdot 24}{2} = \mathbf{(B) 300}$ possible monograms. (Note: $\binom{n}{r} = {}_n C_r = \frac{n!}{(n-r)!r!}$.)
- The largest weight that can be measured is $1 + 3 + 9 = 13$ pounds. One can find combinations for all integers from 1 to 13 (examples: $1=1$, $2=3-1$, $3=3$, $4=3+1$, $5=9-3-1, \dots$), so there are **(B) 13** different weights that can be measured.
Note: In general, the numbers of the form $a_0 3^0 + a_1 3^1 + \dots + a_n 3^n$, where $a_i = 0, \pm 1$ for all $1 \leq i \leq n$, can take on all integer values from $-(1 + 3 + \dots + 3^n)$ to $1 + 3 + \dots + 3^n$. See **balanced ternary** for more.
- At a single point, at most three faces can be seen. Each face has 11^2 cubes, the intersection between any two adjacent faces is 11 cubes, and the intersection between three faces is 1 cube. Using PIE, we see that the union of cubes of the three faces is $(11^2 + 11^2 + 11^2) - (11 + 11 + 11) + 1 = \mathbf{(D) 331}$.
- Let the four digit number be $abcd$. We can have $(a, d) = (a, a + 2)$ for $1 \leq a \leq 7$, or $(a, d) = (a, a - 2)$ for $2 \leq a \leq 9$. This gives $7 + 8 = 15$ choices for a and d . After choosing a and d , we have 8 choices for b , then 7 choices for c , so our answer is $15 \cdot 8 \cdot 7 = \mathbf{(C) 840}$.
- Let a_n be the amount of regions the plane is divided into by n lines. If we have n lines, adding the $n + 1$ -th line gives $n + 1$ extra regions, one for the line splitting the plane, and one for each intersection between the line and the n preexisting lines. Thus, $a_{n+1} = a_n + n + 1$. We have $a_0 = 0$, so we can compute $a_1 = 2, a_2 = 4, a_3 = 7, a_4 = 11, a_5 = 16$, and $a_6 = \mathbf{(C) 22}$.
Alternatively, we see that along with our original region, each line creates an additional region, and each intersection between lines creates another additional region. There are n lines, and $\binom{n}{2}$ intersections (any pair of two lines gives an intersection), so we have $a_n = 1 + n + \binom{n}{2}$.
- The amount of ways to change the attribute of material, size, color, and shape are 1, 2, 3, and 3, respectively. We count all ways to change two attributes: $1 \cdot 2 + 1 \cdot 3 + 1 \cdot 3 + 2 \cdot 3 + 2 \cdot 3 + 3 \cdot 3 = \mathbf{(A) 29}$.
- Each person on the table “covers” at most 3 spots: his own spot, and the two spots adjacent to him. (This count is overlapped when two people are one or two spots away from each other). All 60 spots must be covered, so there must be at least 20 people. This is possible when every third seat is taken, so the answer is **(B) 20**.
- In an n -gon, there are $\binom{n}{2}$ lines that can be drawn between the n points, since each pair of 2 points corresponds to a line. However, n of these lines are sides, not diagonals. Thus, an n -gon has $\binom{n}{2} - n$ diagonals, and plugging in $n = 100$ gives $\frac{100 \cdot 99}{2} - 100 = \mathbf{(A) 4850}$.
- Each pair of two people corresponds to a handshake, so if there are n people, then there are $\binom{n}{2}$ handshakes total. Since $\binom{8}{2} = \frac{8 \cdot 7}{2} = 28$, our answer is **(D) 8**.