



Math Olympiad and Problem Solving Programs  
G210 - Introductory Math Olympiad  
Problem Set 1.2 - Set Theory and Counting Solutions

Name:

Date:

1. There are  $\boxed{25}$  primes total, namely 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 79, 83, 87, and 97.
2. These are the integers from 51 to 99 inclusive, of which there are  $99 - 51 + 1 = \boxed{49}$  total.
- 3.
4. The units digit must be 0, 2, 4, or 6, giving 4 choices total. The tens digit can be any of the given digits except 0, giving 6 choices total. Multiplying these together gives  $\boxed{24}$  choices total.
5. Remember that the letters in alphabetical order are A, E, H, M, S. The first  $4! = 24$  have A as the first letter, and the next groups of 24 have E and H as the first letter, respectively. After these initial 72 words, the next 12 arrangements will start with M. The problem now reduces into a subproblem of finding the 12th permutation of A, E, H, S. Similar to before, the first  $3! = 6$  will start with A, and the next 6 will start with E. The last permutation starting with E will be the 12th one, and this is ESHA. Thus, the 72nd word is MESHA, whose last letter is  $\boxed{\text{(A) A}}$ .
6. The palindromes will be in the form  $aba$ , where  $a, b$  are digits. We must have  $2 \leq a \leq 4$ , so there are 3 choices for  $a$ . Multiplying this by the 10 choices for  $b$ , our answer is  $\boxed{30}$ .
7. We have two possible permutations of "OE" to go first, and of the remaining letters CNTST, we can arrange these in  $\frac{5!}{2!} = 60$  ways, since there are two indistinguishable T's. Multiplying these gives  $\boxed{\text{(B) 120}}$  total permutations.
8. Let the four digit number be  $abcd$ . We can have  $(a, d) = (a, a + 2)$  for  $1 \leq a \leq 7$ , or  $(a, d) = (a, a - 2)$  for  $2 \leq a \leq 9$ . This gives  $7 + 8 = 15$  choices for  $a$  and  $d$ . After choosing  $a$  and  $d$ , we have 8 choices for  $b$ , then 7 choices for  $c$ , so our answer is  $15 \cdot 8 \cdot 7 = \boxed{\text{(C) 840}}$ .
9. (a) Since 6 students did not watch any channels, we have that 34 students watched at least one channel. Then, by PIE, we have  $34 = 16 + 18 + 17 - (9 + 8 + 7) + x$ , where  $x$  is the amount of students who watched all three channels. Solving, we have  $x = \boxed{7}$ .  
(b) Of 9 students who watched ABC and CBS, we have 7 who watched all three networks. Thus, the other  $\boxed{2}$  students watched ABC and CBS, but not NBC.  
(c) We now consider only the students who watched NBC. We have 17 students total, 8 who watch ABC, 7 who watch CBS, and 7 who watch ABC and CBS. By PIE, we have that the amount of students who watched ABC or CBS is  $8 + 7 - 7 = 8$ , so the other  $\boxed{9}$  students watched NBC only.
10. (a) 45% of students played football and soccer, and of these, 5% played all three sports. Thus, the other  $\boxed{40\%}$  of students played football and soccer but not basketball.  
(b) With a method similar to the above, we can find that 10% played soccer and basketball only, and 20% played football and basketball only. Adding these all up, we have  $40\% + 10\% + 20\% = \boxed{70\%}$  of students playing exactly two sports.



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- (c) Using PIE, we have that the percentage of students who played at least one sport is  $70 + 60 + 45 - (45 + 15 + 25) + 5 = 95$ . Thus, only 5% of students do not play any of the three sports.