1. $A \cap B = \{2, 4\}$, so $\boxed{|A \cap B| = 2}$. $A \cap C = \{\}$, so $\boxed{|A \cap C| = 0}$. $A \cup B = \{1, 2, 3, 4, 6\}$, so $\boxed{|A \cup B| = 5}$. $A \cup C = \{1, 2, 3, 4, 7, 8\}$, so $\boxed{|A \cup C| = 6}$.

2. (a) $A \cap B = \{2, 4, 6\}$, and $\{2, 4, 6\} \cap C = \boxed{\{2\}}$. Also, $B \cap C = \{2, 3, 5\}$, and $A \cap \{2, 3, 5\} = \{2\}$.

   (b) $A \cup B = \{1, 2, 3, 4, 5, 6, 8\}$, and $\{1, 2, 3, 4, 5, 6, 8\} \cup C = \boxed{\{1, 2, 3, 4, 5, 6, 7, 8, 11\}}$. Also, $B \cup C = \{1, 2, 3, 4, 5, 6, 7, 11\}$, and $A \cup \{1, 2, 3, 4, 5, 6, 7, 8, 11\} = \{1, 2, 3, 4, 5, 6, 7, 8, 11\}$.

   (c) $B \cup C = \{1, 2, 3, 4, 5, 6, 7, 11\}$ (You've already calculated this!), and $A \cap \{1, 2, 3, 4, 5, 6, 7, 11\} = \boxed{\{2, 4, 6\}}$. Also, $A \cap B = \{2, 4, 6\}$, $A \cap C = \{2\}$, and $\{2, 4, 6\} \cup \{2\} = \{2, 4, 6\}$.

   (d) $B \cap C = \{2, 3, 5\}$, and $A \cup \{2, 3, 5\} = \boxed{\{2, 3, 4, 5, 6, 8\}}$. Also, $A \cup B = \{1, 2, 3, 4, 5, 6, 8\}$, $A \cup C = \{2, 3, 4, 5, 6, 7, 8, 11\}$, and $\{1, 2, 3, 4, 5, 6, 8\} \cap \{2, 3, 4, 5, 6, 7, 8, 11\} = \{2, 3, 4, 5, 6, 8\}$.

   Note: Each of the pair of expressions are equal in general.

3. (a) Add one to each element of this set to get the set of even numbers less than or equal to 20, which has $\lfloor \frac{20}{2} \rfloor = \boxed{10}$ elements.

   (b) This set has $\lfloor \frac{20}{3} \rfloor = \boxed{6}$ elements.

   (c) This set is equal to $\{1, 3, 5, 6, 7, 9, 11, 12, 13, 15, 17, 18, 19\}$, which has $\boxed{13}$ elements. Note: It might be easier to calculate $|A|, |B|, |A \cap B|$ first, then use the Principle of Inclusion-Exclusion.

   (d) This set is all odd multiples of 3 under 20, or $\{3, 9, 15\}$, which has $\boxed{3}$ elements.

4. (a) Let A be the set of girls that can do shorthand typing, and let B the set of girls that can use the word processor. We have $|A| = 11, |B| = 12$, and $|A \cup B| = 18$. By PIE, we have $18 = 11 + 12 - |A \cap B|$, so $|A \cap B| = \boxed{5}$.

   (b) We want to minimize $|A \cup B|$, which we can do by maximizing $|A \cap B|$. We have $|A \cap B| \leq |A|, |B|$, since it is a subset of both, so $|A \cap B| \leq 11$. In the equality case, we have 11 girls who can shorthand type and use the word processor, 1 girl who can only use the word processor, and $\boxed{6}$ girls who have none of these skills.

5. We see that X must contain $\{1, 2\}$, and may contain any subset of $\{3, 4, 5\}$. Also, since $X \subseteq \{1, 2, 3, 4, 5\}$, X cannot have any more elements. There are $\boxed{\textbf{(D)}\ 8}$ subsets of $\{3, 4, 5\}$, each of which corresponds to a possible sets that X can equal.

6. The second statement tells us that 31 girls are brunettes, so 19 girls are blondes. Now, of the 19 blondes, 14 have blue eyes, so 5 blonds have brown eyes. Of the 18 people with brown eyes, we have 5 blondes, so we have $\boxed{\textbf{(E)}\ 13}$ girls with brown eyes who are brunettes.

7. Let x, y, z be the amount of people in sets X, Y, Z, respectively. Since the average age of $X \cup Y$ is 29, we have $29(x + y) = 37x + 23y$, by equating the sums of ages of both sets. This simplifies to $\frac{3}{4}y = x$. Similarly, looking at $Y \cup Z$ gives us $33(y+z) = 23y+41z$, or $\frac{5}{4}y = z$. The total sum of ages is $37x + 23y + 41z$, and the total amount of people is $x + y + z$. Substituting x and z in terms of y, we calculate the average to be $\frac{37x+23y+41z}{x+y+z} = \frac{37(3/4)y+23y+41(5/4)y}{(3/4+1+5/4)y}$. Simplifying this fraction gives $\boxed{\textbf{(E)}\ 34}$.

8. Let us instead find the number of positive integers less than 1000 divisible by 5 or 7, then subtract this answer from 999 to get our final answer. Let A be the set of multiples of 5 less than 1000, and B be the set of multiples of 7 less than 1000. We have $|A| = \lfloor \frac{999}{5} \rfloor = 199, |B| = \lfloor \frac{999}{7} \rfloor = 142$. The set $A \cap B$ is the multiples of 35 less than 1000, and the cardinality of this set is $\lfloor \frac{999}{35} \rfloor = 28$. We have $|A \cup B| = |A| + |B| - |A \cap B| = 199 + 142 - 28 = 313$. Our answer is then $999 - 313 = \boxed{\textbf{(B) } 686}$.

9. Let R, S, H denote the sets of rugby, soccer, and hockey players, respectively. We are given $|R \cup S \cup H| = 136, |R| = 67, |S| = 56, |H| = 40$, $|R \cap S| = 11, |S \cap H| = 12, |R \cap H| = 9$. By PIE, we have $|R \cup S \cup H| = |R| + |S| + |H| - |R \cap S| - |S \cap H| - |R \cap H| + |R \cap S \cap H|$. Plugging in, we have $136 = 67 + 56 + 40 - 11 - 12 - 9 + |R \cap S \cap H|$, or $|R \cap S \cap H| = \boxed{5}$.

10. Let $T_n = \frac{n(n+1)}{2}$, the nth triangular number. Clearly, the nth set has n elements, the first of which is $T_{n-1} + 1$, and the last of which is T_n. Since the elements of the set lie in arithmetic sequence, we can find the sum by multiplying the amount of terms by the average of the first and last term. This gives us

$$\begin{aligned} S_n &= \frac{n(T_{n-1} + 1 + T_n)}{2} \\ &= \frac{n(\frac{(n-1)n}{2} + \frac{2}{2} + \frac{n(n+1)}{2})}{2} \\ &= \frac{n(n^2+1)}{2} \end{aligned}$$

Plugging in $n = 21$, we have $S_{21} = \frac{21(21^2+1)}{2} = \boxed{\textbf{(B) } 4641}$.