



Math Olympiad and Problem Solving Programs
F130 - Advanced Problem Solving
Problem Set 29.1 - Remainders

Name: _____

Date: _____

When we divide a number N by a divisor d , we get a quotient q and a remainder r . In other words, $N \div d = q Rr$. We can rearrange these letters to form the equation:

$$N = d \cdot q + r \quad \text{or} \quad N - r = q \cdot d.$$

Note: many of these problems are very difficult for the 5th grade level, and some problems require algebra. If you got 3 or 4 correct, that is adequate.

1.

2. $7104 \times 519 = 3,686,976$. Divided by 11 gives us

3. Any of the following:

4. Write each of them in the equation given on the top of the page:

$$215 = d \times q + r$$

$$86 = d \times p + r.$$

We are given that the divisor d is the same and the remainder r is the same for both. However, their quotients p and q are different. Let's subtract the bottom equation from the top equation:

$$215 - 86 = dq + r - dp - r \Rightarrow 129 = dp - dq = d(p - q).$$

This means that we need to find some numbers that multiply together to 129. $129 = 3 \times 43$. This is the only prime factorization. So if $d(p - q) = 3 \times 43$, then $d = 3$ or $d = 43$. Let's test these divisors.

$215 \div 3 = 71 R2$, $86 \div 3 = 28 R2$. So when both numbers are divided by , they both have remainder 2.

$215 \div 43 = 5 R0$, $86 \div 43 = 2 R0$. So when both numbers are divided by , they both have remainder 0.

5. Referring to the equation at the top of this page, we can write the number 109 as $109 - 4 = q \cdot d = 105$, or 105 is the product of two whole numbers. How many ways can 105 be written as a product of 2 whole numbers? $1 \times 105, 3 \times 35, 5 \times 21, 7 \times 15$. So how many two-digit numbers divided into 109 will leave a remainder of 4? If $d = 35$, then the quotient will be 3 and remainder 4. If $d = 21$, the quotient will be 5 and the remainder 4. If $d = 15$, the quotient will be 7 and the remainder will be 4. So there are such divisors.

6. What is the remainder when 2008 is divided by 4? 0. So what is the remainder of $2008 \times 2008 \cdots \times 2008$ divided by 4?

7. Referring to the equation at the top of this page, we can write the number 5122 as $5122 - 66 = 5056 = q \cdot d$, where d is some divisor and q is a quotient. Let's see what pairs of whole numbers multiply to 5056.

$$2 \times 2528, 4 \times 1264, 8 \times 632, 16 \times 312, 32 \times 158, 64 \times 79$$



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The problem asks for a two-digit divisor. So let's test what happens when we divide any of the two digit factors listed above into 5122:

$$5122 \div 16 = 320 \text{ R}2.$$

$$5122 \div 32 = 160 \text{ R}2.$$

$$5122 \div 64 = 80 \text{ R}2.$$

$$5122 \div 79 = 64 \text{ R}66.$$

The only one that produces a remainder of 66 is the last. This is because it is greater than the remainder. Notice that 16, 32, 64 are all less than 66, but 79 is greater than 66. So the two-digit number that gives a remainder of 66 when divided into 5122 is $\boxed{79}$

8. We are given that $m + n = 1088$. We are also given that $m = n \times 11 + 32$, which is in the form of the equation at the top of the page. We will have to do a little bit of algebra to solve this problem. Since we know that $m = 11n + 32$, we can substitute this value into the m in the equation $m + n = 1088$. Let's do that substitution: $(11n + 32) + n = 1088$. Now let's simplify. What is 11 n 's plus 1 n ? $12n$. So $11n + n = 12n$. We write $12n + 32 = 1088$. Subtract 32 from both sides and we get $12n = 1056$. So $n = 88$. Since $m + n = 1088$, and $n = 88$, then m must be 1000. $\boxed{m=1000, n=88}$

9. Let's find a pattern in remainders. I will divide by 7 because there are 7 days in a week.

$$10^1 = 10 \div 7 \text{ has remainder } 3.$$

$$10^2 = 100 \div 7 \text{ has remainder } 2.$$

$$10^3 = 1,000 \div 7 \text{ has remainder } 6.$$

$$10^4 = 10,000 \div 7 \text{ has remainder } 4.$$

$$10^5 = 100,000 \div 7 \text{ has remainder } 5.$$

$$10^6 = 1,000,000 \div 7 \text{ has remainder } 1.$$

The remainder pattern will repeat for every power of 10. The pattern has 6 numbers in it. So we divide 2009 by 6: $2009 \div 6 = 334 \text{ R}5$. This means that, as we keep raising 10 to a new power, the remainders when we divide by 7 will go through 334 cycles of 6, and have 5 terms of the pattern left over. The 5th term in the pattern is 5.

So $10^{2009} \div 7$ will have remainder 5. This means that if today is Wednesday, 10^{2009} days from today will be the same day of the week as 5 days from today. 5 days from today is $\boxed{\text{Sunday}}$.

10. This problem has very heavy algebra. Please skip this problem if you don't understand this explanation.

We write each number in the form $N = qd + r$: $171 = dq + r$, $117 = d(q - 3) + r$. We write $q - 3$ because the instructions tells us that the resulting quotient is 3 less than the correct quotient. Now we simplify by subtracting the second equation from the first: $171 - 117 = dq + r - d(q - 3) - r \Rightarrow 54 = dq - dq + 3d = 3d$. Since we know $54 = 3d$, then the dividend d must be $\boxed{18}$