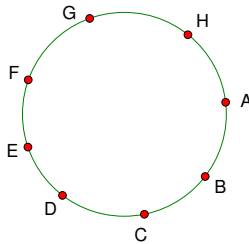


1. $\boxed{3214}$
2. $\boxed{60}$
3. Let's pretend Tony has 4 different hats: his 3 different hats and No Hat. No Hat means he does not wear a hat that day, because the problem says his outfit MAY include a hat, so he can wear a hat or he can not wear a hat. Now we do the multiplication rule to see how many outfits he has. He can wear 4 different shirts AND 5 different pants AND 4 different hats: $4 \times 5 \times 4 = \boxed{80}$ outfits.
4. Let P mean penny, N mean nickel, and D mean dime. Then we have three different values if we use only 1 coin, P, D, or N. If we use 2 coins, we have three different values, P+D, D+N, or P+N. If we use all three coins, then we have one more different value. So altogether there are $3 + 3 + 1 = \boxed{7}$ values.
5. $\boxed{24}$
6. The problem does not say that we can't repeat digits, so we assume that we can repeat digits. How many choices for the first digit? 4 (since we can't have 0 be the first digit). How many choices for the second digit? 5. How many choices for the third digit? 5. So altogether we have $4 \times 5 \times 5 = \boxed{100}$ numbers.
7. $\boxed{48}$
8. Here is our circle.



How many ways are there to select the first point in my triangle? 8. How many ways are there to select the second point for my triangle? 7. How many ways are there to select the third point for my circle? 6. So there are $8 \times 7 \times 6 = 336$ ways to choose a triangle.

BUT! Consider this! Consider I choose points A, D, and F as my points for the triangle. What is the difference between triangle ADF and triangle FDA? Nothing! So there are repeat triangles in our counting. If we choose any three points, how many repeats are there? ADF, AFD, DAF, DFA, FAD, FDA. So for each selection of 3 points, there are 6 repeat triangles. So we need to divide our counting by 6 to take away the repeats.

$336 \div 6 = \boxed{56}$ different triangles.



Math Olympiad and Problem Solving Programs

F130 - Advanced Problem Solving

Problem Set 28.2 - Counting

Name:

Date:

9. Name the 18 people A, B, C, and so on. How many handshakes does A have? He shakes hands with B, C, D, and so on, so he makes 17 handshakes. How many handshakes does B have? We've already counted his handshake with A, so he shakes hands with C, D, and so on, so he makes 16 handshakes. This pattern will continue down to the last pair of people who have 1 handshake. So we need to add $17 + 16 + 15 + \cdots + 3 + 2 + 1 = \boxed{153}$
10. First, let's figure out how we can have four single-digit numbers sum to 34. We know that $34 = 9 + 9 + 9 + 7$ and $34 = 9 + 9 + 8 + 8$. We need to figure out how many different arrangements of the digits 9, 9, 9, 7 and 9, 9, 8, 8 there are.
- For 9, 9, 9, 7, there are 4 arrangements: 9997, 9979, 9799, 7999.
- For 9, 9, 8, 8, there are 6 arrangements: 9988, 8899, 9898, 8989, 9889, 8998.
- In total, there are $4 + 6 = \boxed{10}$ numbers.