Avid Academy
for Gifted Youth

Math Olympiad and Problem Solving Programs

F130 - Advanced Problem Solving

Problem Set 28.1 - The Sum and Product Rules

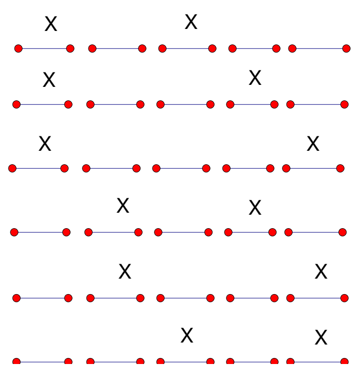
Name: Date:

Remember this rule: AND = multiply, OR = add

1. How many ways are there to choose a ball for Box #1? 6. How many ways are there to choose a ball for Box #2? 5, because we used one in Box 1. How many ways are there to choose a ball for Box #3? 4. Since we choose a ball for Box 1 AND Box 2 AND Box 3, we multiply: $6 \times 5 \times 4 = \boxed{120}$ arrangements.

2. $5 \times 4 \times 3 = \boxed{60}$

3. First, let's figure out the different seating arrangements if Jack and Drew do not sit next to each other in a row of 5. In the diagram, an X represents Jack or Drew, and a blank space represents one of the other three students.

   So there are 6 arrangements where the boys are at least one seat apart. Now let's assign people to this seating arrangement. How many ways are there to assign Jack and Drew to the X spots? Two, Jack and Drew or Drew and Jack. How many ways are there to assign the other three students (1, 2, and 3) to the other three spots? 6: $3 \times 2 \times 1$.

   Finally, we have enough information to find our final computation. We multiply 6 $\textit{seat arrangements} \times 2$ $\textit{boy arrangements} \times 6$ $\textit{other student arrangements} = \boxed{72}$ different arrangements.

4. Let's name each student A, B, C, D, E, F, G, H, I, and J. First let's consider how many games A plays. He plays B, C, D, and so on to J. A plays 9 games. Now consider how many games B plays. He has already played A, so he plays C, D, E, and so on to J. So B plays 8 games. We continue this pattern, so C plays 7 games, D plays 6, and so on until we get to I plays 1. So there are $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$ games, which is $\boxed{45}$ games.

5. First, let's choose the last digit. We want an odd number, so the last digit must be odd. We can choose 3, 5, or 7 for the last digit, because those are our only choices for odd numbers. So we have **3** choices for the last digit.

   Now let's choose the first digit. We have 6 numbers we can choose from, but we've already used one for the last digit, so now we have **5** choices for the first digit.

   Now let's choose the middle digit. We had 6 numbers to choose from, but we used one each for the last and first digits, so now we have **4** choices for the last digit.

   We choose a first AND a middle AND a last digit, so we multiply: $5 \times 4 \times 3 = \boxed{60}$

6. Let's choose the first digit. There are **9** digits we can choose for the first digit (1 through 9). Now that we've chosen the first digit, we have automatically chosen the last digit. Since palindromes are the same forwards and backwards, the first and last digit must be the same. So there is only **1** choice for the last digit.

   Now let's choose the second digit. There are **10** digits we can choose (0 through 9). Now that we've chosen the second digit, we automatically have the fourth digit. So there is **1** choice for the fourth digit.

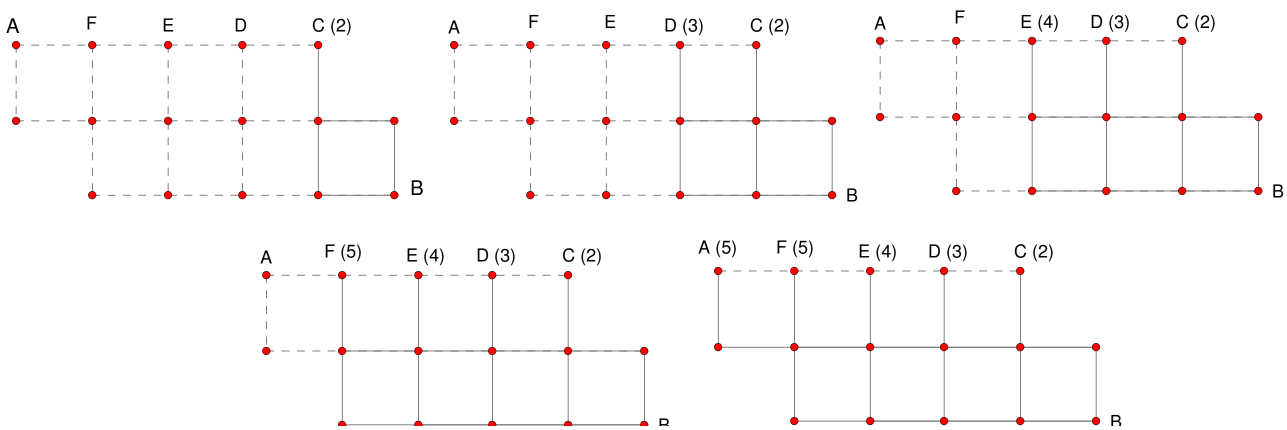
   Now let's choose the middle digit. We have **10** choices.

   Now we multiply: $9 \times 10 \times 10 \times 1 \times 1 = \boxed{900}$ 5-digit palindromes.

7. How many ways can we choose the hundreds digit? **10** choices. Now that we've chosen the hundreds digit, the tens digit is the same as the hundreds digit, so it has **1** choice. Now we choose the units digit, which has **10** choices for the digits. Finally, we multiply $10 \times 1 \times 10 = \boxed{100}$

8. Let's name the 6 stations between SD and LA A, B, C, D, E, and F. Let's say a person gets on the train at SD. Then he can get off at 7 stations: A, B, C, D, E, F or LA. So there are **7** stations. Let's say a person gets on the train at station A. Then he can get off at **6** stations: B, C, through LA. If he gets on at station B, he has **5** choices for stations to get off at. And so on, until if he gets on at F, he has **1** place he can get off the train (LA). A person can get a SD to D ticket OR a B to F ticket OR a SD to LA ticket, etc. So we add: $7 + 6 + 5 + 4 + 3 + 2 + 1 = \boxed{28}$ different tickets.

9. Count by pieces. First, let's pretend the ant goes from point A to C. How many ways are there to get to B from C? 2. Now let's pretend the and goes from point A to D and then goes downward. How many ways are there to get from D to B? 3. And so on...



   From E to B there are 4 ways. From F to B there are 5 ways. From A to B (without going through points C through F, but going down immediately from A), there are 5 ways. In total, there are $5 + 5 + 4 + 3 + 2 = \boxed{19}$ ways to get from A to B.

10. How many ways to color the first square? 4. How many ways to color the second square? 3. To color the third? 2. TO color the last? 1. Multiply: $4 \times 3 \times 2 \times 1 = \boxed{24}$