



Math Olympiad and Problem Solving Programs
E210 - Introductory Math Competitions
Problem Set 26.2 - Arithmetic

Name:

Date:

1. There are 4 ways that 36 could be the third number. Since the two operations are doubling the number and subtracting 12, and we need to work backwards, we will halve the number (or divide by 2) and add 12.

(a) $36 \rightarrow \text{halve the number} = 18 \rightarrow \text{halve the number} = 9.$

(b) $36 \rightarrow \text{halve the number} = 18 \rightarrow \text{add 12 to it} = 30.$

(c) $36 \rightarrow \text{add 12 to it} = 48 \rightarrow \text{halve the number} = 24.$

(d) $36 \rightarrow \text{add 12 to it} = 48 \rightarrow \text{add 12 to it} = 60.$

Now let's check our work by considering what happens when we start with each of the numbers in the four cases, following Zan's rules.

(a) If we start with 9, $9 \leq 25$, so we double it to 18. 18 is also less than 25, so we double it again to 36.

(b) If we start with 30, $30 > 25$, so we subtract 12 to get 18. 18 is less than 25, so we double it to 36.

(c) If we start with 24, $24 \leq 25$, so we double it to 48. $48 > 25$, so we subtract 12 to get 36.

(d) If we start with 60, $60 > 25$, so we subtract 12 to get 48. $48 > 25$, so we subtract 12 to get 36.

So we see that each of these four starting numbers will give us a third number of 36. The problem asks for the sum of these four numbers, so $9 + 30 + 24 + 60 = \boxed{123}$

2. $\boxed{16}$

3. The scale gives us the dimensions of length. For instance, if the scale is $\frac{1}{10}$, then 1 inch in length on the drawing is 10 inches in real life. If the length is 3 inches in the drawing, it is 30 inches in real life. But this is only for length, NOT area. We need to find the scale for area. Let's imagine a square area of 10 inches by 10 inches in real life. This area is 100 in^2 . But on the drawing, its lengths will be 1 in by 1 in, which has an area of 1 in^2 . So for area, the scale is $\frac{1}{100}$.

Since the scale drawing has an area of 3 in^2 , the actual area is $3 \times 100 = \boxed{300 \text{ in}^2}$.

4. $\boxed{4}$

5. Remember the average formulas? One of them is $total = average \times number$. So the total points he earns is the average times the number of tests, or $total = 57 \times 5 = 285$. We can find the highest score possible by pretending the other four tests were the lowest scores possible. If he scored at least 50, then let's pretend he scored 50 on the four of the tests. Then he would have earned 200 points for those four tests. So the maximum number of points he could have scored is $285 - 200 = \boxed{85}$.

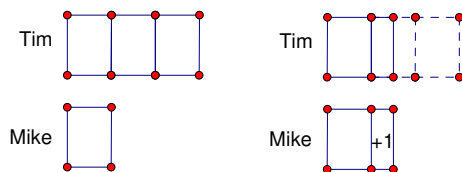
6. First, add the decimals: $.375 + .4 = .775$. Now we write this as a fraction and reduce (a common fraction is just a regular, simplified fraction):

$$.775 = \frac{775}{1000} \text{ (divide out 25 from top and bottom)} = \boxed{\frac{31}{40}}$$

7. This question is asking for the fraction where the distance between satellite and earth is in the numerator, and the distance between earth and the moon is in the denominator. First, let's rewrite $2.4 \times 10^5 = (2.4 \times 10) \times 10 \times 10 \times 10 \times 10 = (24 \times 10) \times 10 \times 10 \times 10 = 240 \times 10 \times 10 \times 10 = 240 \times 1000$.

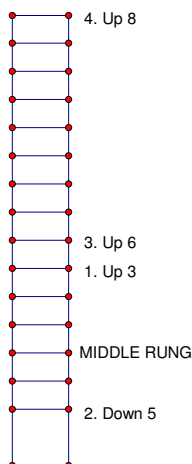
Now let's write the fraction: $\frac{240}{240 \times 1000}$ (cancel the 240 from top and bottom) = $\boxed{\frac{1}{1000}}$

8. Rather than finding each of the amounts of each month, let's do some creative math. The first month, Bob gets \$1000. The second, he gets $\$1000 + 20$. The third, he gets $\$1000 + 2 \cdot 20$. The fourth, he gets $\$1000 + 3 \cdot 20$. And so on, until the last month, where he gets $\$1000 + 11 \cdot 20$. So he gets 12 \$1000's, which is \$12,000. Now we must add $20 + 2 \cdot 20 + 3 \cdot 20 + \dots + 11 \cdot 20$. Let's factor out 20, and so we have $20(1 + 2 + 3 + \dots + 10 + 11)$. You should memorize that $1 + 2 + 3 + \dots + 9 + 10 = 55$; then we know that $1 + 2 + 3 + \dots + 10 + 11 = 55 + 11 = 66$. So the sum is equal to $20 \times 66 = 1320$. So in total, he earns $\$12,000 + \$1,320 = \boxed{\$13,320}$
9. Let's use box diagrams! First, Tim has three times as many coins as Mike, so we give Mike 1 box and Tim 3 boxes. Then, if Mike gets 1 more coin, he will have the same as half of Tim's coins. So we draw this in our diagram.



We can see from the second diagram that Mike's $box + 1$ is equal to Tim's $1\frac{1}{2} boxes$. This must mean that $half - box = 1$, so a whole $box = 2$. So Mike has 2 coins since he has 1 box, and Tim has 6 coins since he has 3 boxes, and altogether they have $\boxed{8}$

10. Let's draw a picture of what is happening.



I have marked each step where we stop. We can see from the image that there are 12 rungs above the middle rung. So in total, there are $12 top rungs + 1 middle rung + 12 bottom rungs = \boxed{25}$ rungs.