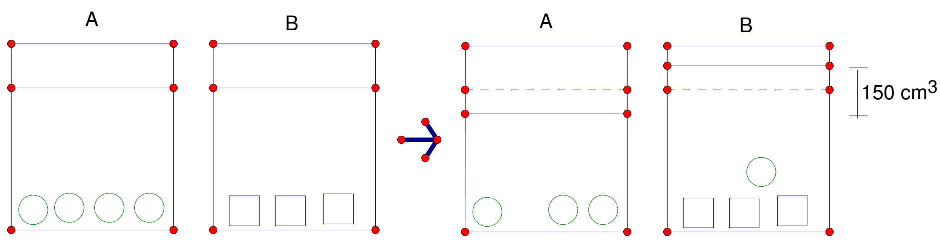
1. Let's create an illustration. First we have jars A and B which are have the same amount of water and the same water level. Then we move a bead from A to jar B, and the difference in volumes is 150 cm$^3$.




From the first diagram, we know that 4 $beads = 3\ blocks$. From the second diagram, we know that the volume of a bead is the change in volume in each jar. Since the total change is 150, each jar changes by $150 \div 2 = 75$. So Jar A loses 75 cm$^3$ of volume, and Jar B gains 75 cm$^3$ of volume, for a total difference of 150. So we know that $bead = 75$. So then 4 beads have a volume of $4 \times 75 = 300$ cm$^3$. We know that 3 $blocks = 4\ beads = 300$, so each block must be $\boxed{100 \text{ cm}^3}$.

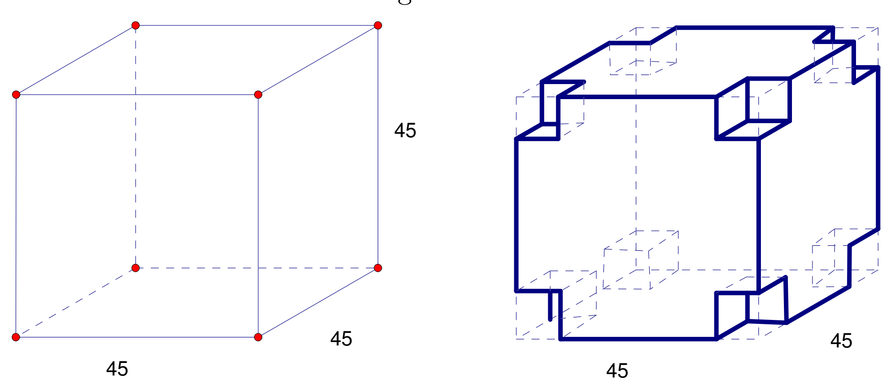
2. The old water level was 3 cm, and the new water level is 12 cm. So the difference in height is $12-3 = 9$ cm. To find the volume of water, we multiply $length \times width \times height = 35 \times 10 \times 9 =$ $\boxed{3150 \text{ cm}^3}$

3. We can picture this container as tall and skinny. There is sand up to height of 7, and we know that the container is half filled with sand and water. That means the sand and water combined are half of the height 34, or $34 \div 2 = 17$. Since the sand takes up 7 cm of the height, the other 10 cm must be water. So there is $14 \times 25 \times 10 =$ $\boxed{3500 \text{ cm}^3}$ water.

4. We can fill in $14 - 9 = 5$ cm more, and $22 \times 18 \times 5 =$ $\boxed{1980 \text{ cm}^3}$

5. A cube has all the same side lengths. So the volume of the cube is $length \times width \times height =$ $45 \times 45 \times 45 = 91125$.

Now we cut a little block of side length 5 from each of the cubes corners. How many corners does a cube have? EIGHT! See the diagram below.




So we need to subtract away the volume of 8 cubes with side length 5. The volume of such a cube is $5 \times 5 = 125$. So the volume of 8 cubes is $125 \times 8 = 1000$. So we subtract $91125 - 1000 =$ $\boxed{90125 \text{ cm}^3}$

6. We have a volume of $40 \times 25 \times 47 = 47000$ cm$^3$ = 47 L in the container. If we pour out 25 L, we have $47 - 25 = 22$ L (or 22,000 cm$^3$) left. We need to find the height of the new water level. We know that its length and width are 40 and 25. Now we need the height. So we know that $40 \times 25 \times \textit{height} = 22000$ cm$^3$. We divide $22000 \div 40 = 550$, and then divide $550 \div 25 =$ $\boxed{22 \text{ cm}}$

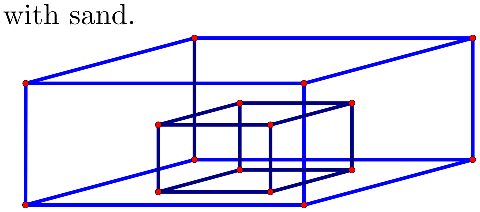
7. We are given that, in beaker A, $2 \textit{ beads} + 1 \textit{ block} + \textit{water} = 520$ cm$^3$, and in beaker B, $1 \textit{ bead} + 2 \textit{ blocks} + \textit{water} = 566$. We are also given $\textit{water} = 420$, so we can make our equations $2 \textit{ beads} + 1 \textit{ block} + 420 = 520$ cm$^3$, and $1 \textit{ bead} + 2 \textit{ blocks} + 420 = 566$. We can simplify these to $2 \textit{ beads} + 1 \textit{ block} = 100$ cm$^3$, and $1 \textit{ bead} + 2 \textit{ blocks} = 146$.

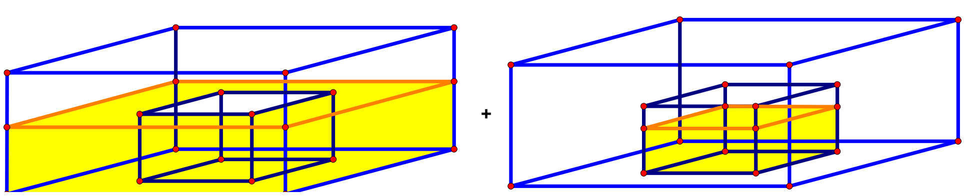
   Now we must solve these equations. You can do it by guessing and checking, or we can consider the following. If I have 1 bead and 2 blocks, the total volume is 146. If I double the number of beads and blocks, so I have 2 beads and 4 blocks, it will double the volume, so it will have 292. So now I have $2 \textit{ beads} + 4 \textit{ blocks} = 292$. I know also that $2 \textit{ beads} + 1 \textit{ block} = 100$. So I rearrange my 2 beads and 4 blocks in two groups: $(2 \textit{ beads} + 1 \textit{ block}) + (3 \textit{ blocks}) = 292$. I can replace the $(2 \textit{ beads} + 1 \textit{ block})$ with 100, so I have $(100) + (3 \textit{ blocks}) = 292$. Now I know that $3 \textit{ blocks} = 192$, so each block is $192 \div 3 =$ $\boxed{64 \text{ cm}^3}$

8. The change in water is $274 - 130 = 144$ L. But we want the change in water level, not the change in volume. So we need to find some height where $120 \times 80 \times \textit{height} = 144000$ cm$^3$. $120 \times 80 = 9600$. So we have $9600 \times \textit{height} = 144000$. So $\textit{height}$ is $144000 \div 9600 =$ $\boxed{15 \text{ cm}}$

9. If $\textit{length} = 14$, and if the block is 4 times as wide as it is long, then $\textit{width} = \textit{length} \times 4 = 14 \times 4 = 56$. If it is one seventh as high as it is wide, then $\textit{height} = \frac{1}{7} \times \textit{width} = 56 \div 7 = 8$. Then the volume of the block is $\textit{length} \times \textit{width} \times \textit{height} = 14 \times 56 \times 8 =$ $\boxed{6272 \text{ cm}^3}$

10. When you have a problem that is complex like this, it is best to draw a nice diagram so you can make sure you are understanding what is going on. First, the top diagram shows how the small box goes into the larger box. Then we need to consider how much sand is used. Consider the bottom to images. First, we must consider the sand that goes in the inner box. Then we need to add to that the sand that goes in the outer box. However, we must keep in mind that the small box takes up some spaces inside the large box, so it is not completely filled with sand.





First let's find the volume of sand used in the small box. It's length and width are 15 and 10, and the sand is 7 high, so the volume of sand in the small box is $15 \times 10 \times 7 = 1050$ cm$^3$.

Now let's find the volume of sand used in the large box. We will find this by finding the volume of sand pretending there is no small box in the way, and then by subtracting the volume of the small box. The volume of sand in the large box if there was no small box is $30 \times 20 \times 10 = 6000$. The volume of the small box is $15 \times 10 \times 10 = 1500$. So the total volume of sand in the larger box is $6000 - 1500 = 4500$.

Now we add the total amount of sand, which is $1050 + 4500 = \boxed{5550 \text{ cm}^3}$