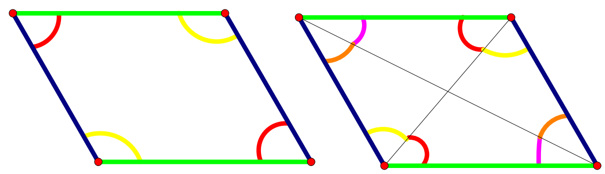
Math Olympiad and Problem Solving Programs

F130 - Advanced Problem Solving

Problem Set 24.1 - Quadrilaterals

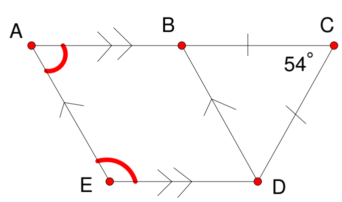
Name: Date:

Things to remember about parallelograms:



- Opposite sides are parallel and have equal length. In the diagram above, the green sides are parallel and equal, and the blue sides are parallel and equal.
- Opposite angles are equal. In the diagram on the left, the two red angles have the same measure, and the two yellow angles have the same measure.
- Two angles touching the same side add up to 180. So yellow angle + red angle = 180.
- In the case where there are diagonals in the parallelogram, like in the right diagram, the opposite angles are the same. The angles marked red, yellow, orange, and pink in the diagram have the same measure as each other.
- In a rhombus, all sides are equal, so blue = green. They are also parallel. When there are diagonals in a rhombus, then the interior angles are the same. So red = yellow and pink = orange in the second diagram.

1. The first step to solving quadrilateral problems is to mark what is given on the diagram. We use arrow marks to signify parallel lines, and we use straight marks to signify lines of the same length. So for this problem, we mark the parallel lines in the parallelogram, and we mark the lines that are the same. Then we mark the angles we are looking for.

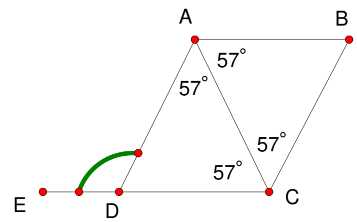


Now we start what we are given and we try to "chase" the angles we are looking for. Let's start with triangle BCD. Since the sides of the triangle are equal, it must be isosceles. We know that the two angles in an isoceles triangle are the same, so we know $\angle CBD = \angle CDB$. The angles in a triangle must add up to 180, and since $\angle C = 54°$, we know that the other two angles in $\triangle BCD = 180 - 54 = 126°$. Since the two angles are the same, they must both be $126 \div 2 = 63°$. So $\angle CBD = \angle CDB = 63°$.

Now we can find $\angle ABD$ because two angles along a straight line add up to 180. Since $\angle CBD = 63°$, $\angle ABD = 180 - 63 = 117°$. Now we can find the angles we are looking for. We know that opposite angles in a parallelogram are the same, so $\angle ABD = \angle AED$, so $\angle AED = \boxed{117°}$.

We also know that two angles on the same side of a parallelogram add up to 180, so $\angle BAE = 180 - \angle AED = 180 - 117 = \boxed{63°}$

2. Since $ABCD$ is a rhombus, we know that all four angles are 57, as marked below.



Since the angles along the same side of a parallelogram must equal 180, we know that $\angle BCD + \angle ADC = 180$. Since $\angle BCD = 57 + 57 = 114$, then $\angle ADC = 66$. Since $\angle ADE + \angle ADC = 180$ since they are on a straight line, $angleADE = 180 - \angle ADC = 180 - 66 = \boxed{114}$

3. $ADBE$ is a trapezoid, so we know that two angles along parallel edges add up to 180. So $\angle AEC + \angle ECB = 180$. Since $\angle AEC = 132$, then $\angle ECB = 180 - 132 = 48$. Since $\angle BCE$ and $\angle ECD$ are along a straight line, they add up to 180, so $\angle BCE + \angle ECD = 48 + \angle ECD = 180$. This means $\angle ECD = 132$. Since $\triangle ECD$ is isosceles, then we know that $\angle E$ and $\angle D$ are the same. Since the angles in a triangle add up to 180, we know $\angle E + \angle D + \angle C = 180$. Since $\angle C = 132$, $\angle E + \angle D = 48$. Since the two are the same, $\angle D = \angle E = 24$. So $\angle AED = \angle AEC + \angle CED = 132 + 24 = \boxed{156}$

4. We can immediately see that $\angle CAD = 180 - 38 - 77 = 65$. We also know that $\angle ADC + \angle DCB = 180$ since they are angles along the edge of a trapezoid. So $\angle BCA = 180 - 38 - 77 = 65$. Since $\triangle ABC$ is isosceles, $\angle B = \angle BCA = 65$. Then we can find $\angle BAC = 180 - 65 - 65 = 50$. So $\angle BAD = \angle BAC + \angle CAD = 50 + 65 = \boxed{115}$.

   ABC is an isosceles triangle where $\overline{AB} = \overline{AC}$ and $ABCD$ is a trapezoid. Find $\angle BAD$.

5. Since $ABEF$ is a parallelogram, then $\angle ABE + \angle BEF = 180$, so $\angle ABC = 180 - 66 - 58 = 56$. Since the shapes are both parallelograms, opposite angles are equal, so $\angle CAB = \angle BEF = 58$, and $\angle ADC = \angle ABC = 56$. We know that $\angle DAB + \angle ABC = 180$, so $\angle DAC = 180 - \angle CAB - \angle ABC = 180 - 58 - 56 = \boxed{66}$

   [Easier way: Since AF and BE are parallel, $\angle ACB = \angle CDE = 66$. Since AD and BC are parallel, $\angle ACB = \angle DAC = 66$.]

6. Since AE and BD are parallel, $\angle BDC = \angle AED = \boxed{87}$