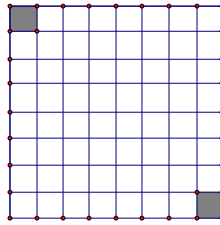
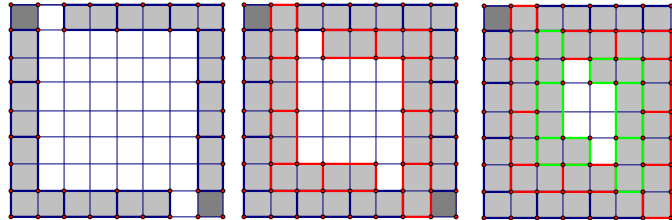


1.
2.
3. The question asks if it is POSSIBLE, not necessary. Of course it is perfectly possible for a two in a group of students to have the same birthday.
4. Let's draw a picture.

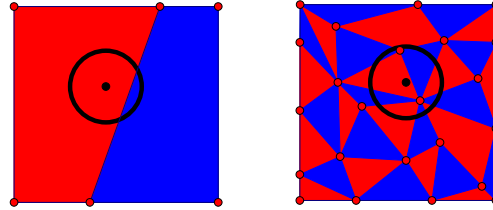


This is the board we start with. We can't immediately tell whether or not we can put dominos on it yet. Let's try to put some dominos in and see what happens.



We can see that there will be no way to put 3 more dominos into the empty space in the middle without cutting the domino. So there is way to do this.

5. $5 + 10 + 15 + 20 + \dots + 90 + 95$. We have *odd + even + odd + even + ... + even + odd*. There are 10 odd numbers (5, 15, 25, ..., 95) and 9 even numbers (10, 20, 30, ..., 90). The 10 odd numbers will have an even sum, and the 8 even numbers will have an even sum, so the whole thing will have an even sum.
6. Let's name all the trees with numbers, so we have Tree 1, Tree 2, Tree 3, and so on until Tree 1,000,000. Since no tree has more than 600,000 leaves, each tree has less than this amount. Let's assign Tree 1 with 1 leaf, Tree 2 with 2 leaves, Tree 3 with 3 leaves, and so on. We stop with Tree 600,000 who has 600,000 leaves. But we still have another 400,000 trees to assign leaf amounts to. We can't give them any different values, because the maximum is 600,000 leaves. So then Trees 600,001 through 1,000,000 must have some amount of leaves less than 600,000, which means there will be two trees in the forest that have the same number of leaves.
7. Let's consider the first question: Is it always possible to find two points of the same color exactly 1 inch apart? Let's look at a few pieces of the plane, one that is simple and one that is complicated. Then we'll draw a circle with radius 1 inch somewhere on the diagram. If the edge of the circle touches the same color as the one at the center, then we have found two pieces with the same color 1 inch apart.



In both images, the center is in red, and part of the circle goes through red. So yes, it is always possible, no matter how simple or complicated the design.

Now we'll consider the second question. Is it always possible to find two points of different colors exactly 1 inch apart? Looking at the diagram again, we see that the centers are both in red, but part of the circle passes through blue. So yes, it is always possible.

8. Let's assign each student to a grade A, B, and C in order.

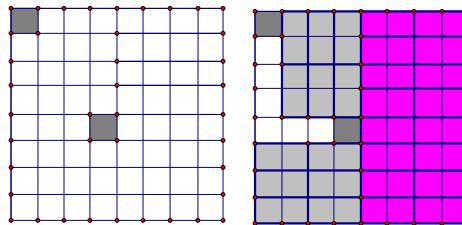
A: 1 student, B: 1 student, C: 1 student. (3 students total)

A: 2 students, B: 2 students, C: 2 students. (6 students total)

A: 3 students, B: 3 students, C: 3 students. (9 students total)

And so on, until A: 8 students, B: 8 students, C: 8 students (24 students total). Now where to put the 25th student? No matter what, if we assign him to A, B, or C, we will end up with at least 9 students getting the same grade.

9. Let's look at the chess board diagram from the previous problem and fill it in piece at a time.



Clearly, we can fill in the last blank spaces with three more dominoes. So yes.

The reason that this problem works while the previous problem doesn't is that we need to be able to split the remaining board pieces up into regions divisible by 4. In this problem, we can split off half the board (shaded purple/pink) which has 32 squares (which is divisible by 4). The two grey regions have 12 squares, which is divisible by 4. The remaining 6 squares can have 3 whole dominoes put in.

10. Free point.