Name: Date:

1. Note: this problem was marked wrong if you didn't write your phone number, because then the TA couldn't check your work.

2. For last digit problems, we find the pattern in units digits:

$$2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 16, 2^5 = 32, 2^6 = 64, 2^7 = 128, 2^8 = 256, \ldots$$

   So we can see the pattern in units digits is $2, 4, 8, 6, 2, 4, 8, 6, \ldots$. The pattern repeats every **4** numbers, so we divide $2008 \div 4 = 502$. There is no remainder, so the units digit of 2^{2008} is the last digit in the pattern, or $\boxed{6}$

3. We repeat the process of the problem above. First we find the pattern in units digits:

$$3^1 = 3, 3^2 = 9, 3^3 = 27, 3^4 = 81, 3^5 = 243, 3^6 = 729 \ldots$$

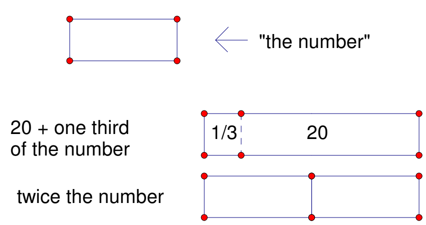
   So we can see the pattern of units digits is $3, 9, 7, 1$. The pattern repeats every 4 numbers, so we divide $2007 \div 4 = 501$ R3. The remainder 3 tells us that the units digit of 3^{2007} is the third in the pattern, or $\boxed{7}$.

4. $1 \times 2 \times 3 \times 4 \times 5 \ldots = (2 \times 5) \times 1 \times 3 \times 4 \ldots = 10 \times 1 \times 3 \times 4 \ldots$, so the units digit of this product will be $\boxed{0}$.

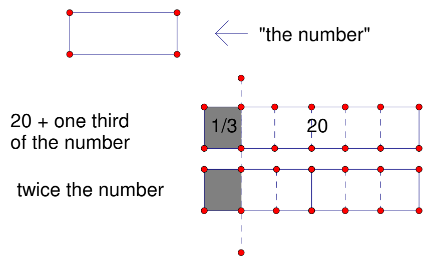
5. $\boxed{24}$

6. $\boxed{\text{Wednesday}}$

7. Use a bar diagram, letting the bar represent "'the number."' We set up a diagram using the information in the problem.



   Now we solve the diagram to find out what "'the number"' is. Notice when we compare the top and the bottom diagrams, we can cut of the $\frac{1}{3}$ from the left end of both diagrams, and we are left with 20 on the top and $1\frac{2}{3}$ of the number on the bottom. Now we can cut the top and bottom into 5 equal pieces, since there are $5\frac{1}{3}$ in the bottom diagram.



If the top is divided into 5 equal pieces, each little box is $20 \div 5 = 4$. So then ”‘the number,”’ which is made of 3 little boxes, is $3 \times 4 =$ $\boxed{12}$

8. Let's manipulate the pattern until we can get a clearer idea of how many numbers there are. As long as we do the same thing to each number, it won't change how many numbers are in the list.

$$2, 5, 8, 11, 14, 17, \ldots, 440, 443, 446, 449 \text{ Add one to each number.}$$
$$3, 6, 9, 12, 15, 18, \ldots, 441, 444, 447, 450 \text{ Divide each number by 3.}$$
$$1, 2, 3, 4, 5, 6, \ldots, 147, 148, 149, 150$$

Clearly, we can see that there are $\boxed{150}$ numbers in this list.

Many of you did this method: subtract the last and first numbers and divide by 3: $(449 - 2) \div 3 = 447 \div 3 = 149$. But you forgot the important step of using this method: add 1! If we do we get $149 + 1 = 150$.

9. $32 \times 162 = (2^5) \times (2 \cdot 3^4) = 2^6 \cdot 3^4 = (2^3 \cdot 3^2)^2 = (8 \cdot 9)^2 = (72)^2$. $\boxed{72}$

10. Organize our counting.

How many times will 3 appear in 1-100? It will appear 10 times in the units digits $(3, 13, 23, \ldots, 93)$ and 10 times in tens digits $(30, 31, 32, \ldots, 39)$. So it will appear 20 times in the first 100 numbers.

How many times will 3 appear in 101-200? It will appear 10 times in the units digits $(103, 113, 123, \ldots, 193)$ and 10 times in tens digits $(130, 131, 132, \ldots, 139)$. So it will appear 20 times in the second 100 numbers.

How many times will 3 appear in 201-299? It will appear 10 times in the units digits $(203, 213, 223, \ldots, 293)$ and 10 times in tens digits $(230, 231, 232, \ldots, 239)$. So it will appear 20 times in the next 100 numbers.

How many times will 3 appear in 300 and 301? 2 times.

So in total we have $20 + 20 + 20 + 2 =$ $\boxed{62}$ 3's.