Important formulas:

$$\textit{total} \div \textit{number} = \textit{average} \qquad \textit{average} \times \textit{number} = \textit{total}$$

Note: this sheet was extremely difficult.

1. When we prime factorize 105, we get $105 = 3 \cdot 5 \cdot 7$. These digits add up to 15, or $3+5+7 = 15$. This means that all of our three digit numbers have the digits 3, 5, and 7. Now let's list the 6 ways we can write a three-digit number with digits adding up to 15 and product equal to 105:

   357, 375, 537, 573, 735, 753.

   Now we average the 6 numbers using the formula: *total* (meaning sum of the 6 numbers) $\div$*number* (or how many elements we added up in the total, so 6) = *average*.

   $\textit{average} = (357 + 375 + 537 + 573 + 735 + 753) \div 6 = 3330 \div 6 = \boxed{555}$

2. Free point

3. Unfortunately, we have to use some algebra to solve this problem. Let's say that the total score of the exams he has taken so far is *sum*. Let's say that he has already taken n exams. So now let's write the average equations given in the problem in terms of *sum* and n.

   If he gets a 48 on his next exam, he will have a total score of $\textit{sum} + 48$, or the total of his previous exams plus the sum of the additional exam. He will also have taken $n + 1$ exams. Then his average will be 56. So we can write

   $$\textit{average} = 56 = \frac{\textit{sum} + 48}{n+1}.$$

   If he gets a 68 on his next exam, he will have a total score of $\textit{sum} + 68$, or the total of his previous exams plus the sum of the additional exam. He will also have taken $n + 1$ exams. Then his average will be 60. So we can write

   $$\textit{average} = 60 = \frac{\textit{sum} + 68}{n+1}(*).$$

   Consider fraction addition for a second. Say we want to add $\frac{3}{7} + \frac{2}{7}$. We can write this as $\frac{3+2}{7} = \frac{5}{7}$. If we wanted, we could also split a fraction into two. Say we want to split $\frac{7}{9}$ into 2 fractions. We could pick $\frac{7}{9} = \frac{3+4}{9} = \frac{3}{9} + \frac{4}{9}$. Since $\frac{3}{9} = \frac{1}{3}$, we can replace it and say $\frac{1}{3} + \frac{4}{9} = \frac{7}{9}$.

   Now let's apply this to the problem. Take $\frac{\textit{sum} + 68}{n+1}$ and split it into 2 by going $60 = \frac{\textit{sum} + 68}{n+1} = \frac{\textit{sum} + 48 + 20}{n+1} = \frac{\textit{sum} + 48}{n+1} + \frac{20}{n+1}$. From the first equation marked (*), we know that $\frac{\textit{sum} + 48}{n+1}$ is equal to 56. So we can replace it in the equation, so now we have $56 + \frac{20}{n+1}$. We know this whole thing equals 60, or $60 = 56 + \frac{20}{n+1}$. Since $60 = 56 + 4$, we know that $\frac{20}{n+1} = 4$. Since $20 \div 5 = 4$, we know $n + 1 = 5$, so n, or the number of exams Mike has already taken, is $\boxed{4}$.

4. When we work with percentages, we must convert them to decimals. To do that, we just move the decimal place 2 spaces to the left. So 22% becomes .22, 47.6% becomes .476. To turn a decimal into a percent, move the decimal two places to the right. So .34 becomes 34%, and .9234 becomes 92.34%.

   First, let's find the total score of the 38 students who were present for the exam. $total = average \times number = .89 \times 38 = 33.82$. So the total percentage that the 38 students earned together is 33.82.

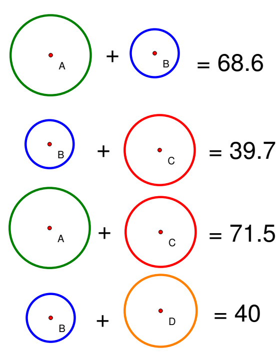
   Now we need to find the average score for the whole class. To find the average, we need to add up all of the scores (the total score of the 38 students and the scores of the 2 absent students) and divide by the size of the whole class, or 40. So $average = \frac{total}{number} = \frac{33.82 + .99 + .99}{40} = \frac{35.8}{40} = .895$. Now we must turn this back into a percent, so we move the decimal two places to the right, and we get $\boxed{89.5\%}$

5. This problem is too hard to explain in this document. If you would like help with this problem, please see the TA before or after class, or on Thursday office hours. $\boxed{15}$

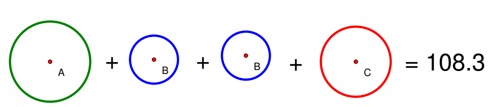
6. To find the total time it took him to walk from A to B, we find the total distance (6000 m) and the average rate. To find the average rate, we average the two speeds he walked at: $\frac{70+80}{2} = \frac{150}{2} = 75$. The distance-time-rate equations are $d = rt, t = d \div r$, and $r = d \div t$. Since we are looking for time, we shall use the middle one: $t = 6000\ m \div 75\ m/min = \boxed{80 \text{ min}}$

7. First, let's find the totals of all the given information. We know $average \times number = total$, so if the average of A and B is 34.3, then $A + B = average \times number = 34.3 \times 2 = 68.6$. Repeating for the other sets of information, we get $B + C = 39.7$, $A + C = 71.5$, and $B + D = 40$.

   Now let's consider this information in a diagram.



   Let's add the first two equations, so we have an A, two B's, and a C, all equaling $68.6 + 39.7 = 108.3$



   Now we know that $A + C = 71.5$, so we can replace the A and C circles with 71.5.

$71.5 + B + B = 108.3$

Therefore, the two B's have to add up to $108.3 - 71.5 = 36.8$. Since they are equal, each B must be $36.8 \div 2 = 18.4$. Now we can find out what the rest of the numbers are. Since $B + D = 40$ and $B = 18.4$, $D = 21.6$. Since $B + C = 39.7$, $C = 21.3$. Since $A + B = 68.6$, $A = 50.2$. Therefore the biggest number is A, and it is $\boxed{A{=}50.2}$

8. Let E be her English grade, let H be her history grade, and M be her math grade. We know that the average of E and H is 91, therefore the total of the two scores is $91 \times 2 = 182$. Also, the average of H and M is 93, so $H + M = 186$, and the average of E and M is 96, so $E + M = 192$. Add up these three equations, and we get $(E+H)+(H+M)+(E+M) = 182+186+192 = 560$. Rearranging, we see that $E+E+H+H+M+M = 560$. Basically, this is the total of double her scores. So we know that $E + H + M = 560 \div 2 = 280$. Now we can find the average because we know the total of her scores: $average = total \div number = 280 \div 3 = \boxed{93.\overline{3}}$

9. This problem was very difficult, so I accepted three answers.

   The first was $\boxed{32,\ 32}$. This is because if you sum up the numbers 1 through 32, you get 528. The average of these 32 numbers is $528 \div 32 = 16.5$. Then if you remove 32 from the list, now the sum is of the numbers 1 through 31, which is 496, and the average is $496 \div 31 = 16$.

   The next answer I accepted was $\boxed{31,\ 16}$. This is because if you average the numbers 1 through 31, you get $496 \div 31 = 16$. Then if you take away 16, your new sum is $496 - 16 = 480$, and the new average is $480 \div 30 = 16$.

   The last answer I accepted was $\boxed{1{,}30}$, which is the most correct answer. This is because if you sum the numbers 1 through 30, you get 465, and the average of these 30 numbers is $465 \div 30 = 15.5$. Then if you remove 1, the new total is 464, and the new average is $464 \div 29 = 16$.

10. This problem is similar to number 3. Let *smallest* be the smallest number, *largest* be the largest number, and *others* be the other 18 numbers. Then we know that $total = smallest + others + largest = average \times number = 13.35 \times 20 = 267$. If the average of *others* increases by 0.15, then the new average is $13.35 + 0.15 = 13.5$. We know that $total = average \times number$, so $total = others = 13.5 \times 18 = 243$. Therefore, we know that $others = 243$. We also know that $smallest + others + largest = 267$. So if we replace *others* with 243 in the second equation, we get $smallest + 243 + largest = 267$. This must mean that $smallest + largest = 267 - 243 = 24$.

    Now we can find the average of the two removed numbers, or *smallest* and *largest*. Their total is 24, and there are 2, so the average is $24 \div 2 = \boxed{12}$