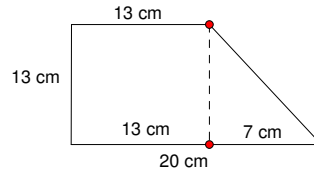


1. $\boxed{539 \text{ in}^2}$

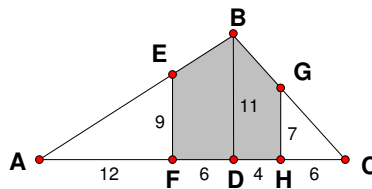
2. The instructions say that the shade is a square and a triangle. So we draw a line that makes a square and a triangle, like in the diagram below. So we find the area of the square: $13 \times 13 = 169$. And we find the area of the triangle: $\frac{1}{2} \times 13 \times 7 = 45.5$. So our total area is $169 + 45.5 = \boxed{214.5}$



3. $\boxed{525}$

4. $\boxed{505}$

5. Refer to the reference points in the diagram below. To find the shaded area, first we will find the area of $EBDF$ and then $BGHD$ and add them together.



To find the area of $EBDF$, we will find the area of triangle ABD and subtract the area of triangle AEF .

$$\text{Area } \triangle ABD = \frac{1}{2} \times (12 + 6) \times 11 = 99$$

$$\text{Area } \triangle AEF = \frac{1}{2} \times 12 \times 9 = 54$$

$$\text{Difference: } 99 - 54 = \mathbf{45} = \text{Area of } EBDF.$$

Similarly, for area of $BGHD$, we will find the area of triangle CBD and subtract the area of triangle CGH .

$$\text{Area } \triangle CBD = \frac{1}{2} \times (6 + 4) \times 11 = 55$$

$$\text{Area } \triangle CGH = \frac{1}{2} \times 6 \times 7 = 21$$

$$\text{Difference: } 55 - 21 = \mathbf{34} = \text{Area of } BGHD.$$

$$\text{So the area of the shaded figure is } 45 + 34 = \boxed{79}.$$

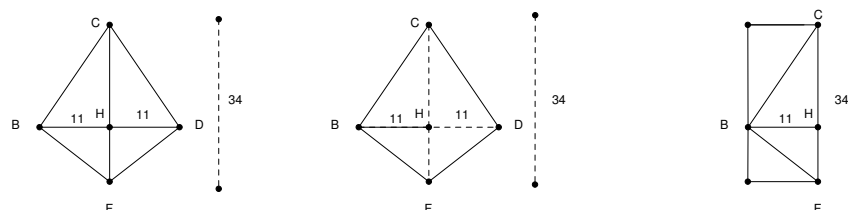
6. Strategy: find the area of $\triangle ACE$ and subtract the areas of triangles CBD and BDF .

$$\text{Area of } \triangle ACE = \frac{1}{2} CG \times AE = \frac{1}{2} 56 \times 40 = 1120.$$

Now we must find the areas of $\triangle CBD$ and $\triangle BDF$. We know that $FG = 22$ and $CG = 56$, so $CF = 56 - 22 = 34$. Now rather than try to find the individual areas of the two triangles, let's do some creative thinking to make calculations easier. Consider the kite shape of $CBFD$. If we cut along the dotted lines shown in the diagram and rearrange the triangles, we get a

rectangle! So we don't need to find the areas of $\triangle CBD$ and $\triangle BDF$ individually, but we find their total area is $11 \times 34 = 374$. So the area of the shaded shape $ABFDEG$ is $1120 - 374 =$

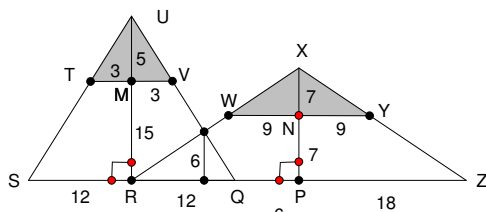
746



7. First, let's fill in information on the diagram from the problem. Since the whole bottom distance $SZ = 48$, and $SZ = SR + RQ + QP + PZ$. We know $SR = RQ = 12$. Since $PZ = RP$, and we know RP is $12 + QP$, $= 12 + 12 + QP + PZ$. So replace these in the addition equation for $SZ = 48 = 12 + 12 + QP + (12 + QP)$. So we know that $48 = 36 + 2QP$'s, so QP must be 6. Thus $PZ = 18$.

Now we know that $UR = 20$, and the snow-top is $\frac{1}{4}$ of UR , so we can determine the heights of the left mountain are split into 5 and 15. Then we know $XP = 14$, and the snow top is $\frac{1}{2}$ of XP , so we can determine the heights of the right mountain are split into 7 and 7.

So now we have all the information drawn onto the image. Now we must find the areas.



The snow tops are the easiest to start with. Area $\triangle UTV = \frac{1}{2} \times 5 \times 6 = 15$, and area $\triangle XWY = \frac{1}{2} \times 7 \times 18 = 63$. So there are $15 + 63 = 78$ square feet of snow top. The glitter paint costs \$2 per square foot, so the cost for the glitter paint is $78 \times 2 = \$156$.

Now let's do the brown paint. Find area of the left mountain by finding the area of $\triangle USQ$ and subtract the area of $\triangle UTV$. Area $\triangle USQ = \frac{1}{2} \times 20 \times 24 = 240$, so the area of left mountain $STVQ = 240 - 15 = 225$. Now we will find the area of the right mountain, BUT we cannot repeat the overlapped area, otherwise we will overestimate the cost of the paint. So we need to find the area of mountain $RWYZ$ and subtract out the little triangle with base RQ and height 6. So to find the area of $RWYZ$, we find the area of $\triangle XRZ$ and subtract $\triangle XWY$. $\triangle XRZ = \frac{1}{2} \times 36 \times 14 = 252$, so area of $RWYZ = 252 - 63 = 189$. Now we need to subtract the area of the little triangle, which is $\frac{1}{2} \times 6 \times 12 = 36$, so we subtract $189 - 36 = 153$.

So there are a total of $225 + 153 = 378$ square feet of brown mountain. The paint costs \$0.25 per square foot, so the brown paint will cost $378 \times 0.25 = \$94.50$.

So the total cost of the mural is \$156 (glitter paint) + \$94.50 (brown paint) = \$250.50

8. 8.1 45

8.2 25



Math Olympiad and Problem Solving Programs

F130 - Advanced Problem Solving

Problem Set 12.1 - Triangles

Name:

Date:

8.3. Find the area of the big triangle, and then subtract the areas of the two small non-shaded triangles. Big triangle = $\frac{1}{2} \times 19 \times 10 = 95$. Bottom little triangle = $\frac{1}{2} \times 8 \times 6 = 24$, side little triangle = $\frac{1}{2} \times 9 \times 10 = 45$. So the area of the shaded little triangle is $95 - 24 - 45 = \boxed{26}$