

1. 55

2. From #1, we know that the first 10 consecutive counting numbers is $1 + 2 + 3 + \dots + 10 = 55$. The sum of the first 10 consecutive counting even numbers is:

$$2 + 4 + 6 + \dots + 20 = 2 \times 1 + 2 \times 2 + 2 \times 3 + \dots + 2 \times 10 \quad (1)$$

$$= 2 \times (1 + 2 + 3 + \dots + 10) \quad (2)$$

$$= 2 \times 55 = \boxed{110} \quad (3)$$

Note that line (2) is an application of the distributive property, $a \times (b + c) = a \times b + a \times c$.

3. 9

4. 499

5. 5

6. Our three two-digit numbers and one one-digit number can look like this:

$$\begin{array}{r} \text{A} \text{ B} \\ \text{C} \text{ D} \\ \text{E} \text{ F} \\ + \quad \text{G} \\ \hline 1 \quad 0 \quad 0 \end{array}$$

This means $B + D + F + G = 10$ or 20 . The only way for $B + D + F + G = 10$ is if B, D, F, G = 1, 2, 3, 4 (not in any particular order). Then $A + C + E = 5 + 6 + 7 = 18$ so we get a sum of 190, which is not what we're looking for. That means $B + D + F + G = 20$. There are a number of ways we can add four numbers from 1 to 7 to get 20 so instead let's look at $A + C + E$. If $B + D + F + G = 20$ then we carry a 2 so $A + C + E + 2 = 10$, or $A + C + E = 8$. A, C, E = 1, 2, 5 or 1, 3, 4 (not in any particular order). But we want the largest number possible so we want A, C, E = 1, 2, 5 (since 5 gives the largest 10's digit). Then B, D, F, G = 3, 4, 6, 7. The largest two digit number then is 57.

7. If n is the smallest of the five consecutive even numbers, then the other four even numbers are $n + 2$, $n + 4$, $n + 6$, and $n + 8$. Their sum is 320 so we get the following equation:

$$n + (n + 2) + (n + 4) + (n + 6) + (n + 8) = 320$$

$$n + n + 2 + n + 4 + n + 6 + n + 8 = 320$$

$$n + n + n + n + n + 2 + 4 + 6 + 8 = 320$$

$$(n + n + n + n + n) + (2 + 4 + 6 + 8) = 320$$

$$5n + 20 = 320$$

$$5n = 300$$

$$n = \boxed{60}$$

8. $\boxed{63}$

9. A line runs through two lines so we need all the combinations of two points we can get: 10 possibilities for the first point and 9 possibilities for the second point give us $10 \times 9 = 90$ possible lines. However, this means we count each line twice since, for example, choosing point A first and point B second gives the same line as choosing point B first and point A second. So we have twice as many lines as we need, $90 \div 2 = \boxed{45}$.

10. $10 + 11 + \dots + 30 = 420$. The only numbers from 10 to 30 that will be replaced are 14, 21, and 28 (replaced with 1.4, 2.1, and 2.8). $14 - 1.4 = 12.6$, $21 - 2.1 = 18.9$, and $28 - 2.8 = 25.2$ so the sum is reduced by $12.6 + 18.9 + 25.2 = 56.7$ so our sum is $420 - 56.7 = \boxed{363.3}$.