

1. (a)  $\frac{1}{9} + \frac{1}{153}$

(b)  $\frac{1}{10} + \frac{1}{190}$

2. (a)  $\frac{1}{13} + \frac{1}{156}$

(b)  $\frac{1}{14} + \frac{1}{182}$

(c)  $\frac{1}{15} + \frac{1}{210}$

3. We will write  $\frac{1}{3}$  as the sum of unit fractions,  $\frac{1}{4}$  and  $\frac{1}{12}$ , and then write each unit fraction as another sum of unit fractions:

$$\frac{1}{3} = \frac{1}{4} + \frac{1}{12}$$

$$\frac{1}{3} = \frac{1}{5} + \frac{1}{20} + \frac{1}{13} + \frac{1}{156}$$

4. (a)  $\frac{5}{6}$

(b)  $\frac{7}{12}$

(c)  $\frac{9}{20}$

5. (a)  $\frac{1}{6}$

(b)  $\frac{1}{12}$

(c)  $\frac{1}{20}$

(d)  $\frac{1}{30}$

6. Notice from #5 that  $\frac{1}{n} - \frac{1}{n+1} = \frac{1}{n(n+1)}$ .

$$\begin{aligned} \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \dots + \frac{1}{9 \times 10} &= \frac{1}{2} + \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \dots - \frac{1}{9} + \frac{1}{9} - \frac{1}{10} \\ &= \frac{1}{2} - \frac{1}{10} = \frac{2}{5} \end{aligned}$$

7. Similar to #6,

$$\begin{aligned} \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \cdots + \frac{1}{9900} &= \frac{1}{4 \times 5} + \frac{1}{5 \times 6} + \frac{1}{6 \times 7} + \cdots + \frac{1}{99 \times 100} \\ &= \frac{1}{4} - \frac{1}{100} = \boxed{\frac{6}{25}} \end{aligned}$$

8. Express  $\frac{1}{10}$  as the sum of four distinct unit fractions.  $\boxed{\frac{1}{12} + \frac{1}{111} + \frac{1}{132} + \frac{1}{12210}}$

9. Express  $\frac{3}{4}$  as the sum of distinct unit fractions.  $\boxed{\frac{1}{2} + \frac{1}{7} + \frac{1}{10} + \frac{1}{140}}$

10. Express  $\frac{3}{5}$  as the sum of distinct unit fractions.  $\boxed{\frac{1}{2} + \frac{1}{10}}$