



Math Olympiad and Problem Solving Programs
F130 - Advanced Problem Solving
Problem Set 4.1 - Arrangements

Name:

Date:

1. First consider how many possibilities there are for the first digit. We can use either 1, 2 or 3 (not zero, because a number starting with 0 would be a two-digit number). Consider the cases:

Case 1: The first number is 1 or 3. There are **2** choices for the first digit. Now consider the last digit. The number must be even, so the last digit has to be even. We can use 0 or 2, so there are **2** choices for the last digit. Of our four numbers, we used one for the first digit and one for the last, so we have **2** possibilities left for the middle digit. There are $2 \times 2 \times 2 = 8$ choices for this case.

Case 2: The first number is 2. There is **1** possibility for the first digit. Now consider the last digit. The number must be even, and we've already used 2, so the last digit must be 0. We have **1** possibility for the last digit. Now consider the middle digit. We have used two of the four possible digits, so we have **2** left to choose from. So there are $1 \times 2 \times 1 = 2$ choices for this case.

There are $8 + 2 = 10$ possible numbers. $\boxed{10}$

2. It is easiest to list the numbers. Make sure to organize your work.

Only one type of digit: 111, 999. **2**.

Two types of digits: 110, 101, 990, 909, 119, 191, 911, 991, 919, 991. **10**.

Three types of digits: 109, 901, 190, 910. **4**.

$2 + 10 + 4 = \boxed{16}$

3. A four digit number cannot start with 0. For example, the number 0513 is just 513, which is three digits. Since the question doesn't specify whether or not repetitions are allowed, we accepted two answers.

With repetition. There are **3** possible choices for the first digit (1, 3, or 5). Then there are **4** possibilities for the second digit, **4** for the third, and **4** for the fourth. So there are $3 \times 4 \times 4 \times 4 = \boxed{192}$ numbers.

Without repetition. There are **3** possible choices for the first digit. Then there are **3** choices left for the second digit, and then there are **2** choices left for the third digit, and finally there is only **1** number left for the last digit. So there are $3 \times 3 \times 2 \times 1 = \boxed{18}$ numbers.

4. $4! = \boxed{24}$

5. $2!2! = \boxed{4}$

6. It is easiest to list the possibilities. Organize your work.

One letter: MMM. **1**.

Two letters: MMO, MOM, OMM, MMY, MYM, YMM. **6**.

Three letters: OMY, OYM, MOY, MYO, YOM, YMO. **6**.

There are $6 + 6 + 1 = 13$ possibilities.

Another interpretation of the question is that no letters can be repeated, because the question says *use the letters in the word MOMMY only once*. So in that case, only the the words in the third case above would be allowed, and the answer would be $\boxed{6}$.

7. $5 \times 5 = \boxed{25}$



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8. We'll use listing again:

One bill: \$1, \$5, \$10. **3** amounts.

Two bills: $\$1 + \$5 = \$6$, $\$5 + \$10 = \$15$, $\$1 + \$10 = \$11$. **3** amounts.

Three bills: $\$1 + \$5 + \$10 = \16 . **1** amount.

There are $3 + 3 + 1 = \boxed{7}$ amounts.

9. $3 \times 3 = \boxed{9}$

10. Use listing. $\boxed{19}$