Avid Academy for Gifted Youth

# Math Olympiad and Problem Solving Programs

F120 - Intermediate Problem Solving

Problem Set 28.2 - Review

Name: Date:

1. (a) $\boxed{3}$

   (b) $1999+999\times998 = 1+999+999+999\times998 = 1+(999+999+999\times998) = 1+(1000\times999) = 1+999000 = \boxed{999001}$

2. How many 0's between 1 and 99? There's one in 10, 20, 30, and so on up to 90, which is **9** zeros.

   How many 0's between 100 and 199? There's one in the units digits of 100, 110, 120, and so on up to 190, which is **10** zeros. There's also 0's in the tens digits of 100, 101, 102, and so on up to 109, which is **10** more zeros.

   How many 0's in 200? **2**.

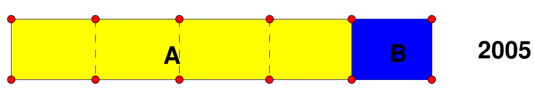
   Now add, and we have $9 + 10 + 10 + 2 = \boxed{31}$

3. This problem is best solved with algebra, which is too difficult to explain at this level. So don't be worried if you got this problem wrong; it was too hard. $\boxed{81}$

4. $\boxed{(4 \times 4 \times 4 - 4) \div 4 = 15}$

5. (a) $\boxed{49}$, $\boxed{36}$ , $\boxed{25}$ (b) $\boxed{16}$ , $\boxed{26}$

6. If $A \div B = 4$, then A is 4 times the size of B. Draw a box diagram.




   We see there are 5 equal boxes that add up to 2005. So each box is worth $2005 \div 5 = 401$. So $A = 4 \times 401 = 1604$, and $b = 401$. Then their difference is $1604 - 401 = \boxed{1203}$

7. First, how many numbers are in this list? They are all the odd numbers from 1 to 101. All the odd numbers from 1 to 99 are half of the numbers from 1 to 100, and since there are 100 numbers from 1 to 100, half of it is 50. So all the odds from 1 to 99 are 50 numbers, so from 1 to 101 its 51 numbers. Now group the list like this:

$$1 - 3 + 5 - 7 + 9 - 11 + \cdots + 97 - 99 + 101$$
$$= 1 + (-3 + 5) + (-7 + 9) + (-11 + 13) + \cdots (-95 + 97) + (-99 + 101)$$

   Now perform each subtraction:

$$= 1 + (2) + (2) + (2) + \cdots (2) + (2)$$

   Now we need to figure out how many 2's are there. We know there were 51 numbers in the list. If we ignore the first number (the 1) and pair up the rest of the numbers, we will have 50 numbers paired up, or 25 pairs. So there are 25 2's, so the sum adds up to $1 + 2 + 2 + \cdots + 2 = 1 + 25 \times 2 = \boxed{51}$

8. Write the sentences in equations like this:

   $3A + 1P = 14O \qquad 6O + 1A = 1P.$

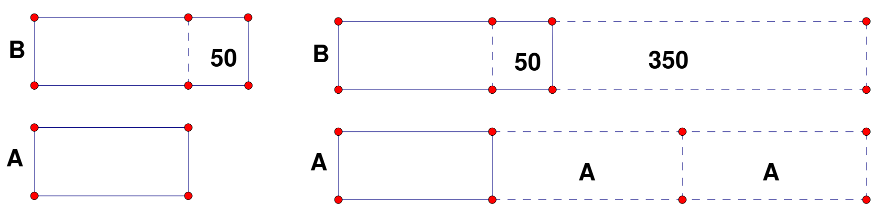
   We want to find what $1P$ is equal to only in terms of oranges. Since we know $6O + 1A = 1P$, we can substitute the $1P$ in the first equation with 6 oranges and 1 apple, or

$$3A + 1P = 14O$$
$$3A + (6O + 1A) = 14O$$
3 apples + 6 oranges + 1 apple = 14 oranges, simplify to 4 apples = 14 - 6 = 8 oranges
$$4A = 8O$$
$$1A = 2O$$

   Since 4 apples = 8 oranges, we can reduce this ratio and determine that 1 apple = 2 oranges.

   Now consider this equation: $6O + 1A = 1P$. If we know $1A = 2O$, we can substitute the $1A$ in the first equation with $2O$. Then we have $1P = 6O + 2O = 8O$. So 1 pear is equal to $\boxed{\text{8 oranges}}$.

9. Draw box diagrams. In the first, we draw a box B that is 50 more than A. In the second, we show that three A boxes is the same as 350 + B.




   Now we solve. We see from the second diagram that an A box + 50 + 350 = 3 A boxes, or A + 400 = 3 A. Then we know that 400 = 2 A, so A must be 200. Then B is 50 more than A, so B is 250. So $A + B = 200 + 250 = \boxed{450}$

10. We use the principle of working backwards to peel away the other numbers layer by layer. If $\dfrac{2005 - 12 \times \bigcirc}{31 \times 11} = 5$, then $2005 - 12\bigcirc = 5 \times 31 \times 11 = 1705$. If the difference between 2005 and $12\bigcirc$ is 1705, then $12\bigcirc = 2005 - 1705 = 300$. Then $\bigcirc = \boxed{25}$.