



Math Olympiad and Problem Solving Programs  
F120 - Intermediate Problem Solving  
Problem Set 28.1 - The Division Theorem

Name:

Date:

1.
2.
3.
4.
5.
6.

7. She records data for the 12th time at 9:00am. She records data for the 11th time five hours before, or 4:00am. She records data the 10th time five hours before that, or 11:00pm. Rather than continue subtracting for each time she records data, let's just figure out how many hours ago she first recorded data.

There are 11 intervals between the 1st time and the 12th time, so the first time she recorded data was  $11 \times 5 = 55$  hours ago. What is 55 hours before 9:00am?  $55 \div 24 = 2 R7$ . So in 55 hours, there are 2 cycles of 24 hours, plus a remainder of 7. So we need to subtract 7 hours from the original start time of 9:00am, which is

8. If a number is divisible by 3, then the digits must add up to a number divisible by 3. If it is divisible by 5, then the last digit must either be 0 or 5. In other words,  $b = 0$  or 5.

Let's start with the smallest possible number. We know the number must be of the form  $C = \overline{5a20}$  or  $D = \overline{5a25}$ . Let's add up the digits so far of  $C$ :  $5 + 2 + 0 = 7$ . In order to make the sum divisible by 3, we need to add 2, so we have  $7 + 2 = 9$ , which is divisible by 3. That would mean we make  $a = 2$ , and  $C = 5220$ . Now let's add up the digits so far of  $D$ :  $5 + 2 + 5 = 12$ . This sum is already divisible by 3, so we do not need to add anything to it, and we can make  $a = 0$ . So  $D = 5025$ . The smallest number between  $C$  and  $D$  is **5025**.

Now let's do the largest possible number. Again, we consider the forms  $C = \overline{5a20}$  or  $D = \overline{5a25}$ , with sums 7 and 12. Now we want to make the sums the largest possible. What is the largest possible number I can add to 7 to make a sum divisible by 3? 8, because  $7 + 8 = 15$ , which is divisible by 3. So in  $C$ , we make  $a = 8$ , so  $C = 5820$ . What is the largest possible number I can add to 12 to make the sum divisible by 3? 9, because  $12 + 9 = 21$ , which is divisible by 3. So in  $D$ , we make  $a = 9$ , so  $D = 5925$ . Clearly, the largest number between  $C$  and  $D$  is **5925**.

Now we find the difference, which is  $5925 - 5025 = \text{$

9. If a number is divisible by 3 and 5, it must be divisible by 15.

How many numbers with four digits or less are divisible by 15? We compute the largest 4-digit number divided by 15, or  $\frac{9999}{15} = 666.\bar{6}$ . Now we round down, and this tells us that 666 numbers less than or equal to 9999 are divisible by 15. But we only want 4-digit numbers, and the 666 includes 2 and 3-digit numbers.



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How many numbers with three digits or less are divisible by 15? We compute the largest 3-digit number divided by 15, or  $\frac{999}{15} = 66.\bar{6}$ . Now we round down, and this tells us that 66 numbers less than or equal to 999 are divisible by 15.

Now we subtract, and we get  $666 - 66 = \boxed{600}$  numbers.

10. Let's make some lists.

The numbers that leave a remainder of 2 when divided by 3 are 2, 5, 8, 11, 14, 17, 20, ...

The numbers that leave a remainder of 4 when divided by 5 are 4, 9, 14, 19, 24, 29, 34, ...

The numbers that leave a remainder of 6 when divided by 7 are 6, 13, 20, 27, 34, 41, 48, ...

As fourth and fifth graders, you don't have the algebraic tools to handle this problem. Unfortunately, the best way to do this problem at this level is brutal force. Here is a screen shot of the Excel document I used to create long lists of the numbers:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI
2	5	8	11	14	17	20	23	26	29	32	35	38	41	44	47	50	53	56	59	62	65	68	71	74	77	80	83	86	89	92	95	98	101	104
4	9	14	19	24	29	34	39	44	49	54	59	64	69	74	79	84	89	94	99	104	109	114	119	124	129	134	139	144	149	154	159	164	169	174
6	13	20	27	34	41	48	55	62	69	76	83	90	97	104	111	118	125	132	139	146	153	160	167	174	181	188	195	202	209	216	223	230	237	244

The smallest number that occurs on all three lists is  $\boxed{104}$ .