Name: Date:

NOTE: squares are rectangles! You must include squares when you count rectangles!

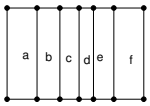
FORMULA: If you have an $M \times N$ grid, then the number of rectangles you can draw is

$$\frac{N(N+1) \cdot M(M+1)}{4}$$

1. There are $\boxed{14}$ squares in figure A. There are 22 rectangles that are not squares, so there are $14 + 22 = \boxed{36}$ rectangles in figure A.

2. $\boxed{8}$

3. Name the individual rectangles as in the diagram below.



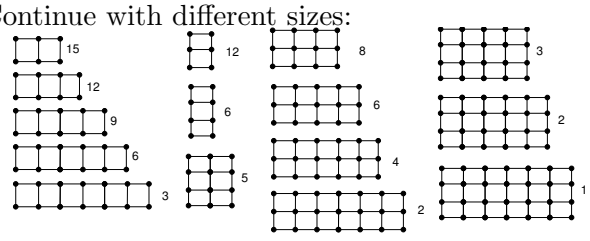
   Now let's count how many rectangles there are. We MUST organize our counting. We start with rectangles that include a, then the rectangles that do not include a, then the rectangles that do not include a or b, and so on.

a	a+b	a+b+c	a+b+c+d	a+b+c+d+e	a+b+c+d+e+f
b	b+c	b+c+d	b+c+d+e	b+c+d+e+f	
c	c+d	c+d+e	c+d+e+f		
d	d+e	d+e+f			
e	e+f				
f					

   So there are $6 + 5 + 4 + 3 + 2 + 1 = \boxed{21}$ rectangles.

4. I will do this two ways. First, I will do the brutal force way. Then I will show you how to use the formula on the top of the page.

   Brutal force way: we count each rectangle of different sizes. First I will count the 1×2 rectangles. Label each of the squares with A, B, C, and so on. Then I can pair together each of the little squares A+B, B+C, C+D, and so on. How many such rectangles are there? 15. Continue with different sizes:



   If we add up all the little numbers, we will get $\boxed{94}$

   Formula way: we use the formula to find out how many rectangles in a 3×6 grid there are. We let $M = 3$ and $N = 6$, and plug in these values into the formula: $\frac{N(N+1) \cdot M(M+1)}{4} =$

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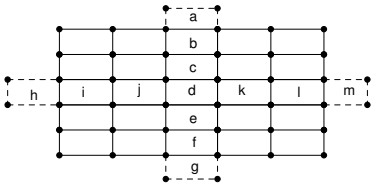
$\frac{6(6+1)\cdot 3(3+1)}{4} = \frac{6\cdot 7\cdot 3\cdot 4}{4} = 126$. But this formula includes the squares. We need to subtract out the number of squares. There are 18 1×1 squares in the grid, 10 2×2 squares in the grid, and 4 3×3 squares in the grid, for a total of $18 + 10 + 4 = 32$ squares. So the total number of rectangles that aren't squares is $126 - 32 = \boxed{94}$.

5. Call the smallest triangle in the figure Little Triangle. There are 9 Little Triangles. There are 3 triangles made of 4 Little Triangles together. Then there 1 large triangle, made of the 9 Little Triangles together. There are a total of $9 + 3 + 1 = \boxed{13}$ triangles.

6. Call the smallest triangle in the figure Little Triangle. There are 8 Little Triangles. There are 4 triangles made of 2 Little Triangles together. There are 4 triangles made of 4 Little Triangles together (half of the square). So there are $8 + 4 + 4 = \boxed{16}$ triangles total.

7. Let's change the image by temporarily removing the extra squares, so all we have is a 5×5 grid, as in the diagram below.



Then we would have a 5×5 square grid, and our formula tells us there would be $\frac{5(5+1)\cdot 5(5+1)}{4} = \frac{5\cdot 6\cdot 5\cdot 6}{4} = 225$ rectangles. Now let's consider how many rectangles are added when we add in each additional square.

If we add square a, then we have 6 new rectangles: a, $a+b$, $a+b+c$, $a+b+c+d$, $a+b+c+d+e$, and $a+b+c+d+e+f$. If we add square g, then we will have 7 new rectangles: g, $g+f$, and so on through $g+f+e+d+c+b+a$. Similarly, if we add square h, we will have 6 new rectangles (h, $h+i$, through $h+i+j+d+k+l$), and if we add square m, we will have 7 new rectangles.

So in total, we have $225 + 6 + 7 + 6 + 7 = \boxed{251}$ rectangles.

8. Use the formula. Here, the grid is 4×5, so let $N = 4$ and $M = 5$. Then we have $\frac{N(N+1)\cdot M(M+1)}{4} = \frac{4(5+1)\cdot 5(5+1)}{4} = \frac{4\cdot 5\cdot 5\cdot 6}{4} = \boxed{150}$ rectangles.

9. Here we want only squares, which are easy enough to count. Notice the figure is basically a 6×6 square grid with 8 additional squares glued on. Let's count the sizes of squares individually.

1×1 squares: $36 + 8 = 44$.

2×2 squares: there are 5 in each bundled row (which is two rows of single squares together), and there are 5 rows, so there are $5 \times 5 = 25$, plus 4 extra ones we can make with the additional 8 small squares. So we have $25 + 4 = 29$.

3×3 squares: $4 \times 4 = 16$.

4×4 squares: $3 \times 3 = 9$.

5×5 squares: $2 \times 2 = 4$.

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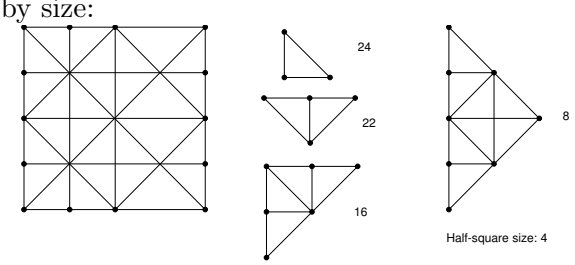
6×6 squares: 1.

There are a total of $44 + 29 + 16 + 9 + 4 + 1 = \boxed{103}$

10. Count by size:



There are $24 + 22 + 16 + 8 + 4 = \boxed{74}$ triangles.

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