



Math Olympiad and Problem Solving Programs

F120 - Intermediate Problem Solving

Problem Set 26.2 - Count Line Segments

Name:

Date:

1. There are two methods for this problem.

Elementary school way: there are 11 points, so there are 10 segments between the points. So there are 10 1-unit segments. Now let's see how many 2-unit segments we can make. We can connect the 1st and 3rd point, 2nd and 4th point, and so on. There will be 9 2-unit segments. Then you count how many 3-unit segments (there will be 8), 4-unit segments (there will be 7), and so on until you have the whole segment (there is just 1). So we add up $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = \boxed{55}$. We will refer to this method throughout the rest of this document as **The Triangle Method**.

Middle school way: we have eleven points on our line, so let's pick one of the points to name A. There are 11 choices of what we can assign as A. Now let's choose a second point to call B. We've already chosen 1 point, so there are 10 points left to name B. So there are $11 \times 10 = 110$ ways to create a line segment AB. But hold on: isn't AB the same as BA? They have the same exact distance, so we've double counted every segment! To undo this overcounting, we divide our multiplication by 2: $110 \div 2 = \boxed{55}$

2. first diagram: The Triangle Method (explained in problem 1) says there are 6 lines, so add $1 + 2 + 3 + 4 + 5 = \boxed{15}$.

second diagram: $\boxed{20}$

third diagram: $\boxed{24}$

3. $1 + 2 + 3 + 4 = \boxed{10}$

4. There are 5 stations. Let's imagine each station to be a point along a line. To count how many tickets there are, we need to count how many segments there are. Using the Triangle Method (explained in problem 1), there will be $1 + 2 + 3 + 4 = 10$ ways to connect two stations, so there are $\boxed{10}$ tickets. (we don't double it to 20 because it asks for how many ticket fares there are, and the price doesn't change depending on the direction the bus is going).

5. This problem was ambiguous, so I allowed a few different answers.

If we consider the 6 stations to include San Diego and Las Vegas, then we need to use the Triangle Method to count $1 + 2 + 3 + 4 + 5 = 15$. So there are 15 different tickets. If you consider a ticket that says "San Diego to Las Vegas" different from a ticket that says "Las Vegas to San Diego," then we need to double 15 to 30.

If we consider the 6 stations to include San Diego but not Las Vegas, then there are 7 stations, then we need to use the Triangle Method to count $1 + 2 + 3 + 4 + 5 + 6 = 21$. So there are 21 different tickets. If you consider a ticket that says "San Diego to Las Vegas" different from a ticket that says "Las Vegas to San Diego," then we need to double 21 to 42.

If we consider the 6 stations to exclude both San Diego and Las Vegas, then there are 8 stations, then we need to use the Triangle Method to count $1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$. So there are 28 different tickets. If you consider a ticket that says "San Diego to Las Vegas" different from a ticket that says "Las Vegas to San Diego," then we need to double 28 to 56.

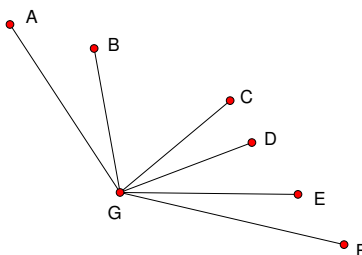
6. Let's separate the diagram into two lines, count the number of segments on each, and add them together.

The line that starts in the top left corner and goes to the bottom right has 8 points, so by the Triangle Method, there are $1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$ segments.

The line that starts in the top right corner and goes to the bottom left has 7 points, so by the Triangle Method, there are $1 + 2 + 3 + 4 + 5 + 6 = 21$ segments.

So in total there are $28 + 21 = \boxed{49}$ segments

7. Label the diagram as shown below.



First, let's count how many possible angles there are in the diagram without considering their measure. There are 6 segments, so there are $1 + 2 + 3 + 4 + 5 = 15$ angles. Now let's manually count with are greater than 90° , or a right angle (if you aren't sure, line up the corner of a paper with the segments and see if the angle is greater than the angle of the paper corner): $\angle AGF, \angle AGE, \angle AGD, \angle BGF, \angle BGE$. So there are 5 that are too large, so we throw them out. So there are $15 - 5 = \boxed{10}$ angles smaller than 90° .

8. Note: this problem is the same as problem 3! $\boxed{10}$
9. This problem is the same as problem 1, only with a much larger number. Referring to problem 1, we consider two methods:

Elementary school method: we know from other problems on this sheet, that since there are 199 points, we need to add $1 + 2 + 3 + \dots + 197 + 198$. Here is a formula for summing all the numbers in a row from 1 to a number: $number \times (number + 1) \div 2$. So we use the formula, replacing *number* with 198 and *number + 1* with $198 + 1 = 199$: $198 \times 199 \div 2 = 39402 \div 2 = \boxed{19701}$.

Middle school method: we have 199 points on our line, so let's pick one of the points to name A. There are 199 choices of what we can assign as A. Now let's choose a second point to call B. We've already chosen 1 point, so there are 198 points left to name B. So there are $199 \times 198 = 39402$ ways to create a line segment AB. But hold on: isn't AB the same as BA? They have the same exact distance, so we've double counted every segment! To undo this overcounting, we divide our multiplication by 2: $39402 \div 2 = \boxed{19701}$