



Math Olympiad and Problem Solving Programs
F120 - Intermediate Problem Solving
Problem Set 24.2 - Payments

Name:

Date:

1. For this sheet, we must carefully organize our counting. I will organize it by changing one dollar amount at a time.

\$5	\$2	\$1
(1)	(4)	(8)
1	1	1
1	0	3
0	4	0
0	3	2
0	2	4
0	1	6
0	0	8

Here, the chart shows the dollar amounts, how many of each we have (in parenthesis below the dollar amounts), and then the list of how many of each we can use to make the proper amount. From the first line, if we use 1 \$5, 1 \$2, and 1 \$1, we will have \$8.

So there are $\boxed{7}$ possible payments.

2. We make a chart.

\$10	\$2	\$1
(2)	(5)	(10)
2	0	0
1	5	0
1	4	2
1	3	4
1	2	6
1	1	8
1	0	10
0	5	10

So there are $\boxed{8}$ such payments.

3. We make a chart.

Basket A	Basket B
3	0
2	1
1	2
0	3

So there are $\boxed{4}$ placements.

4. We need to make 6 with different positive integers. Let's start with how we can make sums with 2 numbers. We can do $1 + 5$ and $2 + 4$. We can't do $3 + 3$ because we need the numbers to be different. Now we find how many sums we can do with 3 numbers. We can do $1 + 2 + 3$, and that's all. We can't do 4, 5, or 6 numbers because we can't use different numbers. So there are only $\boxed{3}$ such sums.

5. $3 \times 2 \times 1 = \boxed{6}$



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6. Let's make a chart symbolizing all the different ways Hunter can take out two coins, and find their sums.

\$0.01 (2)	\$0.05 (2)	\$0.10 (2)	\$0.50 (1)	Sum
2	0	0	0	\$0.02
0	2	0	0	\$0.10
0	0	2	0	\$0.20
1	1	0	0	\$0.06
1	0	1	0	\$0.11
1	0	0	1	\$0.51
0	1	1	0	\$0.15
0	1	0	1	\$0.55
0	0	1	1	\$0.60

So there are 9 different sums.

7. With making different postages, we can use 1 stamp only, 2 stamps, 3 stamps total, and so on up to 7 stamps (because we only have 7 stamps total). So we make a chart, first find out how many postages we can make with just 1 stamp, then 2 stamps, and so on until 7. The amounts below are in cents.

30 (4)	50 (3)	Total
1	0	30
0	1	50
2	0	60
1	1	80
0	2	100
3	0	90
2	1	110
1	2	130
0	3	150
4	0	120
3	1	140
2	2	160
1	3	180
4	1	170
3	2	190
2	3	210
4	2	220
3	3	240
4	3	270

So we count 19 postages.

8. Let's find out how many amounts Michelle H can take with two bills. Then we will find the amount that is \$2 less than Michelle H's amount, and see if it is possible to make that



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amount with three remaining bills. For instance, if Michelle H picks out 2 \$10's, she will have \$20. We need Michelle H to have \$2 more than Michelle K, so we need to see if we can give Michelle K $20 - 2 = \$18$ with the remaining bills. Michelle H has the two tens, so can we make \$18 with the remaining 2 each of \$1, \$2, and \$5? No, because the max we can make is $\$5 + \$5 + \$2 = \12 .

Michelle H's \$				total	\$2 less	can we make this?	remaining
\$1 (2)	\$2 (2)	\$5 (2)	\$10 (2)				
2	0	0	0	2	0	no	X
0	2	0	0	4	2	no	X
0	0	2	0	10	8	no	X
0	0	0	2	20	18	no	X
1	1	0	0	3	1	no	X
1	0	1	0	6	4	no	X
1	0	0	1	11	9	yes: $5 + 2 + 2$	$1 + 5 + 10$
0	1	1	0	7	5	no	X
0	1	0	1	12	10	no	X
0	0	1	1	15	13	yes: $10 + 2 + 1$	$1 + 2 + 5$

There are only two combinations that work. Michelle H can pick out a \$1 and a \$10 for \$11, and Michelle K can pick out a \$2, \$2, and \$5 for \$9. Then Michelle H has \$2 more than Michelle K. The left over bills still in the box are a \$1, \$5, and \$10, which is \$16 total.

In the second case, Michelle H can take out a \$5 and a \$10 for a total of \$15, and Michelle K can pick a \$10, \$2, and \$1 for a total of \$13. Then Michelle H has \$2 more than Michelle K. The remaining bills are a \$1, \$2, and \$5, which is \$8 total.

We are given Dr. Li takes the remaining bills which are \$5 more than Michelle H. In the first case, Dr. Li would have \$16, which is \$5 more than Michelle H's \$11. In the second case, Dr. Li has \$8, which is NOT \$5 more than Michelle H's \$15. So it must be the first case. So the sum of the Michelles' money is $\$11 + \$9 = \boxed{\$20}$

9. We need to guess and check how many stamps he bought. Let's start with 4 40-cent and 5 80-cent (remember we know he bought 9 total). This would cost $4 \times \$0.40 + 5 \times \$0.80 = \$5.60$. This is less than the actual amount, so we need to guess more of the more expensive stamp. Now try 3 40-cents and 6 80-cents. This would cost $3 \times \$0.40 + 6 \times \$0.80 = \$6.00$. This is the right amount. So Mason bought $\boxed{3}$ 40-cent stamps.

10. If Albert pays \$24, the cashier can give \$5 change. $\boxed{\text{pay 12 } \$2 \text{ bills}}$