Name: Date:

The principle idea with these problems is to ORGANIZE your counting!

Also, please note, to count how many numbers are in a list of numbers (for example, $14, 15, 16, \ldots, 76, 77$), you take the last number, subtract the first number, and add 1 (so $77 - 14 + 1 = 63 + 1 = 64$. So there are 64 numbers in that list).

1. It takes 9 digits to print the first 9 pages of the book, 1 - 9. So we have $85 - 9 = 76$ more digits to use. From now on, each number is a 2 digit number, because now we have to print 10, 11, 12, and so on. So let's divide our remaining digits by 2: $76 \div 2 = 38$. So we have 38 more numbers to print. We've printed 9 pages with single digits and another 38 numbers with 2 digits. So we have $9 + 38 = \boxed{47}$ pages.

2. Single digit: It takes 1 digit to print each of the pages 1 - 9. So this uses $1 \times 9 = 9$ digits.

   Two digits: It takes 2 digits to print each of the pages 10 - 80. How many numbers are in this list? The strategy for doing this is to subtract the small number from the large number and add 1: $80 - 10 + 1 = 70 + 1 = 71$. So there are 71 pages with 2 digits. This uses $71 \times 2 = 142$ digits.

   So the total digits are $9 + 142 = \boxed{151}$ digits.

3. Single digits: it takes 9 digits to print 1 - 9. So we have used 9 of our digits, and we have $131 - 9 = 122$ digits left.

   Two digits: We have 122 digits used to print all the double-digit numbers. First, let's divide by 2: $122 \div 2 = 61$. So we have 61 two-digit page numbers.

   So the total number of pages we've printed is $9 + 61 = \boxed{70}$.

4. Single digit: there are 9 one-digit numbers. So we use $1 \times 9 = 9$ digits to print them.

   Two-digit: the two digit numbers go from 10 to 99. To count how many numbers there are, use the formula on the top of the page: $99 - 10 + 1 = 90$. So there are 90 numbers with 2 digits, which uses $90 \times 2 = 180$ digits.

   Three-digit: the three digit numbers go from 100 - 200. Count how many numbers: $200 - 100 + 1 = 101$ numbers. So we use $101 \times 3 = 303$ digits.

   So in total, it takes $9 + 90 + 303 = \boxed{402}$ digits.

5. To organize our counting, we will count by digit.

   Units digit: a "0" appears in the units digit every 10 numbers, like in the page numbers $10, 20, 30 \ldots, 110, 120$. There are 12 numbers in this list.

   Tens digit: a "0" appears in the 10's digit of a number in the 100's (so page $100, 101, 102, \ldots 109$), which has 10 numbers.

   So there are $12 + 10 = \boxed{22}$ zero digits.

6. Units digit: a "1" appears every 10 numbers. So $1, 11, 21, \ldots, 121$. There are 13 numbers in this list.

   Tens digit: a "1" appears in the tens digit in the 10's (so pages $10, 11, 12, \ldots 19$) and the 110's ($110, 111, 112, \ldots 119$), which each have 10 numbers for a total of 20 numbers.

Hundred's digit: a "1" appears in every page number greater than 99, so $100, 101, 102, \ldots 123$. There are $123 - 100 + 1 = 24$ numbers in this list.

So in total, we have $13 + 20 + 24 = \boxed{57}$ ones.

7. There are 20 "3"s in the first 100 pages of the book. This is just a helpful fact to memorize. This is true because there are 10 3's in the units digit $(3, 13, 23, \ldots 93)$, and there are 10 3's in the tens digit $(30, 31, 32, \ldots 39)$. So now we have 4 more 3's to use. The next number with 3 in it is 103. The next number after is 113. The third is 123, and the fourth is $\boxed{130}$.

8. If a book uses 155 digits for its page numbers, then the last page number is in the double-digits. If we take away 8 pages, we take away 8 two-digit numbers, or $8 \times 2 = 16$ digits. So we subtract 16 digits from our total: $155 - 16 = \boxed{139}$.

9. The numbers that have 0's are $10, 20, 30, 40, 50$, so 5 numbers. The numbers that contain 3 in the units are $3, 13, 23, 33, 43, 53$, so 6 numbers. The number that contain a 3 in the tens digit are $31, 32, 34, 35, 36, 37, 38, 39$ (note we already counted 30 in the list of 0's and 33 in the first list of 3's), which is 8 numbers. So there are $5 + 6 + 8 = 19$ numbers with 0's and 3's, so there must be $56 - 19 = \boxed{37}$ numbers without.

10. Single digits: 1 - 9. 9 digits.

    Double digits: 10 - 99. 90 numbers with 2 digits each: $90 \times 2 = 180$ digits.

    Three digits: 100 - 105. 6 numbers with 3 digits each: $6 \times 3 = 18$ digits.

    So there are $9 + 180 + 18 = \boxed{207}$ digits used to print the book.

    Counting 1's: 1 appears in the units digit every 10 numbers (the pages $1, 11, 21, \ldots, 101$), which is 11 numbers. A 1 appears in the tens digit in the 10's $(10, 11, 12, \ldots, 19)$, which is 10 numbers. A 1 appears in the hundreds digit in every number greater than 99 (100, 101, 102, 103, 104, 105), which is 6 numbers. So there are $11 + 10 + 6 = 27$ 1's.

    Counting 5's: 5 appears in the units digit every 10 numbers (the pages $5, 15, 25, \ldots, 105$), which is 11 numbers. A 5 appears in the tens digit in the 50's $(50, 51, 52, \ldots, 59)$, which is 10 numbers. A 5 appears never appears in the hundreds digits. So there are $11 + 10 = 21$ 5's.

    So there are $27 + 21 = \boxed{48}$ 1's and 5's.