



Math Olympiad and Problem Solving Programs
F120 - Intermediate Problem Solving
Problem Set 14.2 - Numbers and Digits

Name: _____

Date: _____

1. We want to replace A and C with a digit (0-9) so that when we round it, we will get the number on the right.

(a) $32A800 \approx 320,000$. So for $32A800$ rounding to $320,000$, we see that the first two digits are the same, so we need to replace A with a smaller number so we will round down. For instance, if $A=0$, then $32A800 = 320,800$, which we would round to $320,000$. If $A=9$, then $32A800 = 329,800$ which we would round to $330,000$. So we want to test all the values of A that would allow us to round down. Those values are $\boxed{A=0,1,2,3,4}$

(b) $5B00 = 5000$ $\boxed{B=0}$

(c) The problem should have said $99C47 \approx 100,000$. Use the same strategy as for part a, and the values of C are $\boxed{C=5,6,7,8,9}$

2. $\boxed{3421}$

3. Our product is 5 digits long, so how long should our multipliers be? Consider $100 \times 100 = 10,000$. We have 2 three digit numbers, and they multiply to a 5 digit number. So we can assume our reverse-digit numbers are three digits. Let's call the number ABC and the reverse-digit number CBA , where A, B , and C are one-digit numbers. Set up a multiplication as shown below, were we will multiply each number one at a time and then add down. Since the last digit of our product is 5, then the multiplication of $A \times C$ must be 5. The only way we can do this is if one of them is 1 and the other is 5. So let $C = 5$ and $A = 1$. Replace these in our multiplication. Now multiply like normal and just write B where we don't know the number, like in the image below. Now we see that when we add down, $B + 5B = 6B$ has a units digit of 6. So what can B be? It can be 1, because $1 + 5 = 6$. So if $ABC = 115$, let's test the multiplication: $115 \times 511 = 58765 \neq 92565$. So $B \neq 1$. What else could B be? It could also be 6, because $6 + 5 \times 6 = 6 + 30 = 36$, which has a units digit 6 and a carry of 3. So now let's test if $ABC = 165$: $165 \times 561 = 92565$. So we have found our numbers. $\boxed{165, 561}$

$\begin{array}{r} ABC \\ \times BA \\ \hline \\ \\ \hline 92565 \end{array}$	$\begin{array}{r} 1 B 5 \\ \times 5 B 1 \\ \hline \\ \\ \hline 92565 \end{array}$
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4. Set up an addition like the one in the problem before. Call our original number $AB7$, where A and B are one-digit numbers. Our new number is created moving the 7 to the front, so now we have $7AB$. Since the new number is 189 greater than the original number, we can write $AB7 + 189 = 7AB$. Now let's add. We start with the units digits, and we see that $7 + 9 = 16$, so we put the 6 in for B and carry the 1. Now we know $6 = B$, so let's replace it in our equation: $A67 + 189 = 7A6$. Now we need to add $6 + 8 + 1 = 15$, where 6 and 8 are the tens digits and 1 is the carry from the units digits. So we get $A = 5$. Let's check our addition is correct one more time: $567 + 189 = 756$. So the original number is $\boxed{567}$

5. Note this problem says BIGGEST to SMALLEST. The first 24 numbers will start with 5, the next 24 numbers (25th - 48th) will start with 4, the next 24 numbers (49th - 72nd) will start with 3, and the next 24 numbers (73rd - 96th) will start with 2. So the 95th number starts with 2, and the rest of the numbers are the second smallest arrangement. So first let's find the smallest arrangement of digits 1, 3, 4, 5: 1345. Now the second smallest just switches the last two digits, so we get 1354. So the 95th number is $\boxed{21354}$
6. $\boxed{1}$
7. If we want to find the maximum number, we need to find the way to add numbers up to 15 so that we use the most numbers. We notice $15 = 5 + 4 + 3 + 2 + 1 + 0$, so we have 6 digits. To make it the maximum number, we write the numbers in descending order, and we get $\boxed{543210}$
8. How can we add up to 12 using 4 different digits? $1 + 2 + 4 + 5$ and $1 + 2 + 3 + 6$. We can arrange each of these sets of 4 digits in 24 ways as the second problem tells us. So we have $24 + 24 = \boxed{48}$ such numbers.
9. Set up a diagram like in problem 3. In the diagram, the boxes with the same color should contain the same number.

$$\begin{array}{r}
 \begin{array}{ccc} \color{yellow}{\square} & \color{cyan}{\square} & \color{magenta}{\square} \\ + & \color{yellow}{\square} & \color{magenta}{\square} & \color{cyan}{\square} \\ \hline
 \end{array} \\
 X \quad 7 \quad 3 \quad Y
 \end{array}$$

Let's start with the hundreds digits, or the yellow boxes. Can a number plus itself ever equal 7? No, so we know that there must be a carry from the tens and units digits over to the hundreds digits. I'll write a 1 over the yellows to remind us that we need a carry. Now let's look at the tens digits. We need a number and another number to add up to 13. We don't want it to add up to 3 because we need the carry for the yellow boxes. So let's think of numbers that add up to 13: $9 + 4$, $8 + 5$, $7 + 6$. Let's try 9 and 4:

$$\begin{array}{r}
 1 \\
 \begin{array}{ccc} \color{yellow}{\square} & \color{cyan}{9} & \color{magenta}{4} \\ + & \color{yellow}{\square} & \color{magenta}{4} & \color{cyan}{9} \\ \hline
 \end{array} \\
 X \quad 7 \quad 3 \quad Y
 \end{array}$$

Then we get $9 + 4 = 13$, so the units digit $Y = 3$, and then we carry the one, but oh no! Now for the tens digits, we have $1 + 4 + 9 = 14$, so our tens digit isn't 3 anymore! This means that instead of adding up to 13, we need to add up to 12. We can add up to 12 in these ways: $9 + 3$, $8 + 4$, $7 + 5$, and $6 + 6$. Now let's test 9 and 3 in our diagram.



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$$\begin{array}{r}
 \\
 1 1 \\
 \color{yellow}{\square} \color{cyan}{9} \color{magenta}{3} \\
 + \color{yellow}{\square} \color{magenta}{3} \color{cyan}{9} \\
 \hline
 X 7 3 Y
 \end{array}$$

It works now. So we get $Y = 2$, carry the 1, $9 + 3 + 1 = 3$ in our tens digits, carry the 1, and now we need to find out what the yellows are. We know that it has to add up to 17, so we get that $1 + \text{yellow} + \text{yellow} = 17$, so yellow must be 8. So now we get $893 + 839 = 1732$. So the question asks how many three digit numbers we can do this with? The yellow digit must always be 8, but the last two digits can be 93, 39, 84, 48, 75, 57, and 66 (or our combinations that add up to 12. Try each of them to make sure you agree with me). So we have 7 numbers that will work.

10. This question was somewhat ambiguous, so I accepted any answer that I could make sense of.

Using 3 digits to make 3 three-digit numbers add up to 1332, here are the various responses I accepted:

$$\boxed{999} + 222 + 111 = 1332$$

$$\boxed{543} + 435 + 354 = 1332$$

$$\boxed{624} + 462 + 246 = 1332$$

$$\boxed{921} + 192 + 219 = 1332$$