



# Math Olympiad and Problem Solving Programs

F120 - Intermediate Problem Solving  
Problem Set 12.2 - Factors and Multiples

Name:

Date:

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- $\boxed{1,3,5,9,15,45}$
- $12 \times 2 = 2 \times (\boxed{4}) \times 3$
  - $12 \times 4 = (\boxed{3}) \times 8 \times 2$
  - $6 \times 14 = 7 \times (\boxed{12})$
  - $35 \times 36 = 35 \times (\boxed{9}) \times 4$
- The factors of 12 are: 1, 2, 3, 4, 6, 12. The factors of 18 are: 1, 2, 3, 6, 9, 18. The ones they have in common are  $\boxed{1,2,3,6}$
- The least common multiple (LCM) of 4 and 5 is just  $4 \times 5 = 20$ . The next multiple of 20 is  $2 \times 20 = 40$ , then  $3 \times 20 = 60$ .  $\boxed{20,40,60}$
- In order for a number to have 2, 3, and 5 as its factors, it must have  $2 \times 3 \times 5 = 30$  as a factor. Only 3 numbers between 1 and 100 have 30 as a factor: 30, 60, and 90. So  $\boxed{3}$
- Prime factorize 124:  $2^2 \times 31$ . The factors are 2 and 31, and their sum is  $\boxed{33}$ .
- $\boxed{28}$
  - $\boxed{35}$
  - $\boxed{49}$
  - The factors of 56 are 1, 2, 4, 7, 8, 14, 28, 56. The ones between 10 and 40 are 14 and 28. 28 is a multiple of 4, so the answer is  $\boxed{14}$
- We need to find the greatest common divisor of 64, 80, and 96. Prime factorize each:  $64 = 2^6$ ,  $80 = 2^4 \times 5$ ,  $96 = 2^5 \times 3$ . The only number they have in common is 2, and the smallest exponent is 4, so the GCD is  $2^4 = \boxed{16}$
- Guess and check. He is 3, next year he is 4, NOT a multiple of 5. He is 6, next year is 7, NOT a multiple of 5. He is 9, next year is 10 (is a multiple of 5), next year is 11 (prime). So he is 9 now in 2009, so he will be  $\boxed{50}$  in 2050.
- BONUS: 2, 6, 6. If you want to know why, ask a TA or Dr. Li.