Math Olympiad and Problem Solving Program

F120 - Intermediate Problem Solving

Problem Set 6.1 - Sum of Patterns

Name: Date:

1. $1+2+3+4+5+6+7+8+9+10+11+12+13+14 = \boxed{105}$

2. $21+22+23+\ldots+40+41+42$. How many numbers are in this sequence? Use the formula: $N = \frac{L-F}{D}+1$, where L is the last number (so $L = 42$), F is the first number (so $F = 21$), D is the difference between the numbers (the numbers increase by 1, so $D = 1$), so $N = \frac{42-21}{1}+1 = \frac{21}{1}+1 = 21+1 = 22$. There are 22 numbers in the sequence. Now we find the sum using the sum formula: $S = \frac{(F+L)\times N}{2}$. Plug all our values in: $S = \frac{(21+42)\times 22}{2} = \frac{63\times 22}{2} = \frac{1386}{2} = \boxed{693}$

3. The first even number is 2, and the 20th even number is 40. Use our sum formula: $S = \frac{(F+L)\times N}{2} = \frac{(2+40)\times 20}{2} = \frac{42\times 20}{2} = \frac{840}{2} = \boxed{420}$

4. $\boxed{155}$

5. $\boxed{250}$

6. $\boxed{450}$

7. $\boxed{165}$

8. On the first day, Ethan read 33 pages. Then on the next ten days, his reading followed the pattern 11 pages, 22 pages, 33 pages... So first, we will find the sum of the last 10 days of reading. $F = 11$, $L = 110$, so $S = \frac{(F+L)\times N}{2} = \frac{(11+110)\times 10}{2} = \frac{121\times 10}{2} = \frac{1210}{2} = 605$. Now we add the first day of reading to the last ten days of reading: $33 + 605 = \boxed{638}$

9. $\boxed{33}$

10. Carefully analyze the pattern: 0 handshakes, 1 handshake, 2 handshakes, ... 9 handshakes. So we sum the numbers $0+1+2+3+\ldots+8+9 = \boxed{45}$