

1. We first count the pages that have a 2 in the units digit of its page number: 2, 12, 22, 32, 42, 52, 62; 7 pages.

Next we count the pages that have a 2 in the tens digit of its page number: 20, 21, 22, 23, 24, 25, 26, 27, 28, 29; 10 pages.

This gives us a total of  $7 + 10 = 17$  pages. However, we counted page 22 twice! So really only 16 pages contain the digit "2" in its page number.

2. It is implied that any time the time has a "4" as one of its digits, it will display an incorrect time.

Each hour, the digit 4 is displayed at :04, :14, :24, :34, :44, :54, which is 6 minutes. It will also be displayed 10 minutes from :40 to :49 for a total of  $10 + 6 = 16$  minutes. However, we subtract 1 because we counted :44 twice. 15 minutes.

We can multiply 15 minutes by 12 since it will be 15 minutes for each of the 12 hours. This gives us a total of  $12 \times 15 = 180$  minutes.

We will also add another 60 minutes during the time from 4:00 to 4:59 to give us  $180 + 60 = 240$  minutes. However, we already counted 15 minutes for the hour from 4:00 to 4:59 previously so we need to subtract this extra time.  $240 - 15 =$ 225 minutes total.

3. Albert

4. Tuesday

5. Justincase

6. We always need one student to row the boat back across the river. This means that each trip we can really only take 2 students across the river. 3 students go across, 2 students get off, and 1 student must row back to starting side of the river. The only exception is the very last trip, at which point all 3 students can leave the boat. We subtract 3 from 13 to get  $13 - 3 = 10$  to get the number of students dropped off prior to the last trip. Then we know that there are  $10 \div 2 = 5$  trips prior to the last trip, giving us a total of  $5 + 1 =$ 6 trips.

7. Head

8. Noticed that our repeating period is  $\clubsuit \diamond \heartsuit \spadesuit$ . Every 4 shapes, we will repeat ourselves. If we have 45 shapes, we repeat our period  $45 \div 4 = 11R1$  times, or really only 11 times. The remainder of 1 tells us that at the end of repeating our period, we start another period and our last, or 45th shape, is the first shape in our pattern; Club:  $\clubsuit$ .

Since we repeat the pattern 11 times, and  $\heartsuit$  appears only once each period,  $\heartsuit$  appears a total of 11 times.



# Math Olympiad and Problem Solving Program

F110 - Introductory Problem Solving

Problem Set 19.1 - Period

Name:

Date:

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9. Similar to lowest common multiple problems, we can count using the larger number. In this case, the larger number is 7. The difference is, we must add the left over candies. Counting in this manner, we get 9, 16, 23, 30, ... While counting, we must look for the first number which has a remainder of 3 when divided by 5. Clearly this is  $\boxed{23}$ .  $9 \div 5 = 1R4$ .  $16 \div 5 = 3R1$ .  $23 \div 5 = 4R3$ .
10. Let's first count the pages that have a "7" in its units digit: 7, 17, 27, 37, 47, 57, 67, 77, 87, 97; 10 pages.
- Next we count the pages that have a "7" in its tens digit: 70, 71, 72, 73, 74, 75, 76, 77, 78, 79; 10 pages.
- This gives us a total of  $10 + 10 = 20$  pages. However, we counted 77 twice so we have to subtract the extra page and we count a total of  $\boxed{19}$  pages.