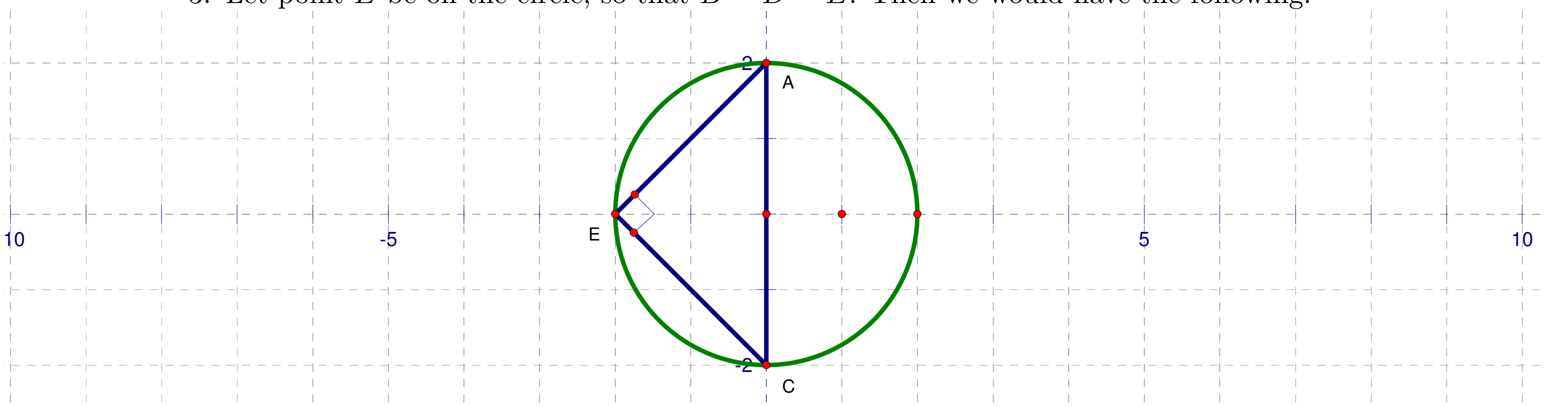
1. $\boxed{10}$

2. $\boxed{15^\circ}$

3. Let point E be on the circle, so that $B = D = E$. Then we would have the following:



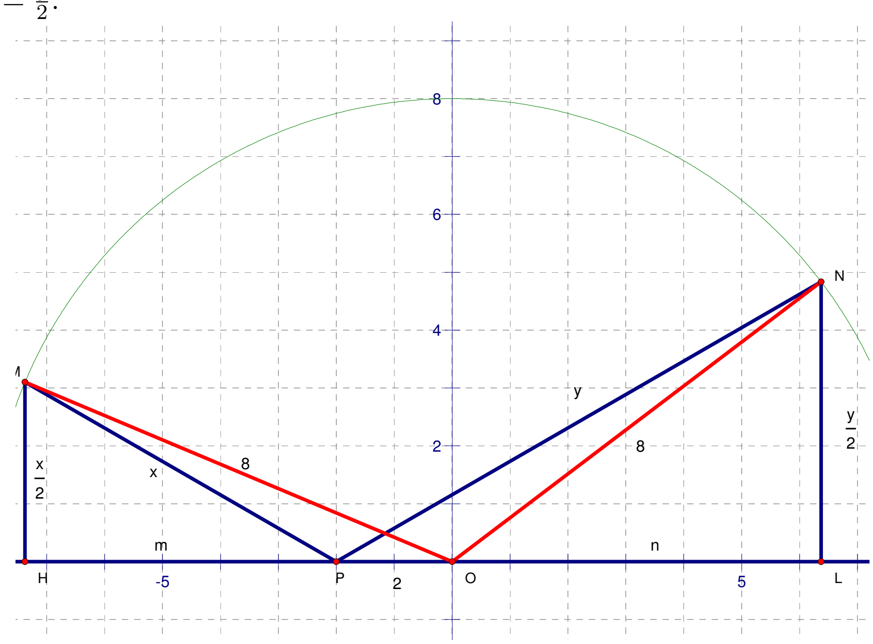
Clearly, $AE^2 + CE^2 = 4^2 = \boxed{16}$

4. $\boxed{75^\circ}$

5. Infinitely many solutions.

6. $\boxed{12}$

7. Observe the fancy diagram. The red lines are radii and have length 8. The length $HP = m$, $MP = x$, $OL = n$, $PN = y$. Notice that since the angle measures are 30°, then $MH = \frac{x}{2}$ and $NL = \frac{y}{2}$.



Now we have a lot of Pythagorean relationships. We have

$$(\frac{x}{2})^2 + m^2 = x^2 \ (1) \qquad (m+2)^2 + (\frac{x}{2})^2 = 8^2 \ (2)$$

$$(\frac{y}{2})^2 + n^2 = 8^2 \ (3) \qquad (n+2)^2 + (\frac{y}{2})^2 = y^2 \ (4)$$

Simplify (1) to $m^2 = \frac{3}{4}x^2 \Rightarrow m = \frac{\sqrt{3}}{2}x$. Substitute this into (2) and solve:

$$\begin{aligned}
(\frac{\sqrt{3}}{2}x + 2)^2 + (\frac{x}{2})^2 &= 8^2 \\
\frac{3}{4}x^2 + 2\sqrt{3}x + 4 + \frac{x^2}{4} &= 64 \\
x^2 + 2\sqrt{3}x - 60 &= 0 \\
x &= \frac{-2\sqrt{3} \pm \sqrt{12 + 4 \cdot 60}}{2} \\
&= \frac{-2\sqrt{3} + \sqrt{252}}{2} \\
&= \frac{-2\sqrt{3} + 6\sqrt{7}}{2} \\
&= 3\sqrt{7} - \sqrt{3}
\end{aligned}$$

Now we repeat the process with (3), which we simplify to $(n+2)^2 = \frac{3}{4}y \Rightarrow n = \frac{\sqrt{3}}{2}y - 2$. Substitute this into (4) and solve:

$$\begin{aligned}
(\frac{\sqrt{3}}{2}y - 2)^2 + (\frac{y}{2})^2 &= 8^2 \\
\frac{3}{4}y^2 - 2\sqrt{3}y + 4 + \frac{y^2}{4} &= 64 \\
y^2 - 2\sqrt{3}y - 60 &= 0 \\
y &= \frac{2\sqrt{3} \pm \sqrt{12 + 4 \cdot 60}}{2} \\
&= \frac{2\sqrt{3} + 6\sqrt{7}}{2} \\
&= 3\sqrt{7} + \sqrt{3}
\end{aligned}$$

Finally, $PN + PM = x + y = 3\sqrt{7} - \sqrt{3} + 3\sqrt{7} + \sqrt{3} = \boxed{6\sqrt{7}}$.

8. $\boxed{\text{No}}$

9. $\boxed{48}$

10. Consider a circle with 6 equidistant points placed on the edge. Then each of those points form an equilateral triangle when connected to the center and to their neighbors. The sides lengths of each of these triangles is the radius r. If we want the distance between any two points to be LARGER than 1994 (the radius), then with 6 points we have a distance of exactly 1994. If we have one less point, then all the distances between two points will always be larger than 1994. $\boxed{n = 5}$