

1. For each of the small shaded triangles, one side is k and the other is $1 - k$ (since the larger square has area 1). Since the shaded area is $\frac{1}{4}$, the area of the smaller square is $\frac{3}{4}$, and the side length of the square is $\sqrt{\frac{3}{4}}$. We can set up a Pythagorean theorem for one of the shaded triangles:

$$\begin{aligned} k^2 + (1 - k)^2 &= \frac{3}{4} \\ k^2 + k^2 - 2k + 1 &= \frac{3}{4} \\ 2k^2 - 2k + \frac{1}{4} &= 0 \\ k^2 - k + \frac{1}{8} &= 0 \end{aligned}$$

From here you can solve for the values of k and multiply, or we can use Vieta's Theorem to determine that the product of the roots is $\boxed{\frac{1}{8}}$

2. First, $[QPR] = \frac{1}{2} \cdot 6 \cdot 12 = 36$. Then $[QPR] = \frac{1}{2} \cdot QS \cdot PR \Rightarrow 36 = \frac{1}{2} \cdot QS \cdot 8$. So $QS = \boxed{9}$
3. The largest square has area 49. The length of the side of the medium square is $\sqrt{2^2 + 5^2} = \sqrt{4 + 25} = \sqrt{29}$. The length of the side of the smallest square is $\sqrt{2^2 + (\sqrt{29} - 2)^2} = \sqrt{37 - 4\sqrt{29}}$. So the area of the smallest square is $37 - 4\sqrt{29}$. The percentage of small area to large area is $\frac{37 - 4\sqrt{29}}{49} \approx \boxed{31.5\%}$
4. $\boxed{\sqrt{3} - \frac{\pi}{2}}$
5. The area of an equilateral triangle with side length s is $\frac{\sqrt{3}}{4}s^2$. Since the area of the truncated equilateral triangle was 60, the side length, or the length of AD , is the solution of $\frac{\sqrt{3}}{4}s^2 = 60 \Rightarrow s^2 = 80\sqrt{3} \Rightarrow s = \sqrt{80\sqrt{3}}$. Since the area of the trapezoid is $\frac{3}{4}$ the area of the truncated equilateral triangle, then BC was a midline of the triangle, so $AC \perp CD$, $BD \perp AB$, $\angle BAN = \angle NAD = \angle ADN = \angle NDC = 30^\circ$. Draw a perpendicular from N to AD , and name the intersection point S . We know $AS = SD = \frac{1}{2}\sqrt{80\sqrt{3}}$. Also, $\triangle ASN$ is 30-60-90, so $SN = \frac{AS}{\sqrt{3}}$, so $SN = \frac{\sqrt{80\sqrt{3}}}{2\sqrt{3}}$. Now we have

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$$\begin{aligned}
 [AND] &= \frac{1}{2} \cdot AD \cdot SN \\
 &= \frac{1}{2} \cdot 2AS \cdot SN \\
 &= AS \cdot SN \\
 &= \frac{\sqrt{80\sqrt{3}}}{2} \cdot \frac{\sqrt{80\sqrt{3}}}{2\sqrt{3}} \\
 &= \frac{80\sqrt{3}}{4\sqrt{3}} \\
 &= \boxed{20}
 \end{aligned}$$

6. $\sqrt{288 + 144\sqrt{3}} = \boxed{23.18}$

7. Free point.

8. $\boxed{3\sqrt{10}}$

9. The area of the square is 36. Consider the area on the left half of the square, which has total area 18. The top and bottom triangles on the left side can be moved together to create a square of side length 3, so the area of the shaded square is **9**. Now consider the third triangle with base length 2. It has height 3, so the area of the triangle is **3**.

Now consider the triangles on the right half of the square. The top right triangle has height and base 3, so its area is **4.5**. The triangle on the bottom right is half the triangle with area 4.5, so it has area **2.25**.

So we have total area $9 + 3 + 4.5 + 2.25 = 18.75 = \frac{75}{4}$. So the fraction of the square that is shaded is $\frac{\frac{75}{4}}{36} = \frac{75}{144} = \boxed{\frac{25}{48}}$

10. $\boxed{36}$