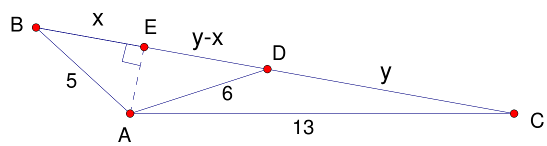
1. $\boxed{12}$

2. First we draw a diagram.



Draw an altitude from A to BC. Let $BD = DC = y$, and let $BE = x$, so $ED = y - x$. Now we can set up a few Pythagorean statements: $AE^2 = 5^2 - x^2$, $AE^2 = 6^2 - (y - x)^2$, $AE^2 = 13^2 - (y + y - x)^2$. Set the first two equal to each other and the second two and simplify.

$$\begin{aligned} 5^2 - x^2 &= 6^2 - (y-x)^2 \\ 25 - x^2 &= 36 - y^2 + 2xy - x^2 \\ 11 &= y^2 + 2xy \quad (1) \end{aligned}$$

$$\begin{aligned} 6^2 - (y-x)^2 &= 13^2 - (2y-x)^2 \\ 36 - y^2 + 2xy - x^2 &= 169 - 4y^2 + 4xy - x^2 \\ 133 &= 3y^2 - 2xy \quad (2) \end{aligned}$$

$$\begin{aligned} (2) - (1) \Rightarrow 2y^2 &= 122 \\ y &= \sqrt{61} \end{aligned}$$

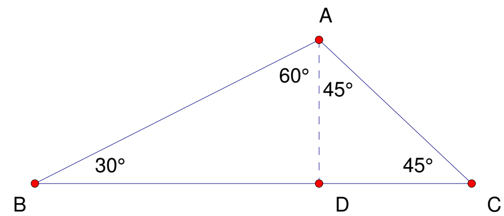
$BC = 2y = \boxed{2\sqrt{61}}$

3. We are given that $\triangle ABC$ is 30-60-90, where $\angle C = 30$ and $\angle B = 60$. Then we know that, since $AB = 1$, $CB = 2$ and $AC = \sqrt{3}$. Use Angle Bisector Theorem. Let $BD = x$, then $CD = 2 - x$.

$$\begin{aligned} \frac{\sqrt{3}}{2-x} &= \frac{1}{x} \\ \sqrt{3}x &= 2 - x \\ (1+\sqrt{3})x &= 2 \\ x &= \frac{2}{1+\sqrt{3}} = \boxed{\sqrt{3}-1} \end{aligned}$$

4. $\boxed{14}$

5. Do NOT use law of cosines or sines to solve this problem!



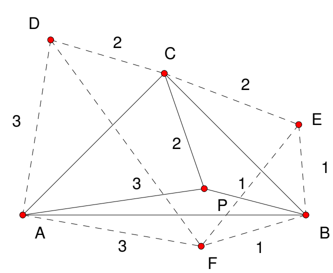
We fill in our information. Since we have a 45-45-90 triangle, let $DC = AD = x$, and then $AC = \sqrt{2}x$. Then $BD = 8 - x$, and since ABD is 30-60-90, $AD = \frac{8-x}{\sqrt{3}}$. Solve the equality for AD.

$$\begin{aligned} \frac{8-x}{\sqrt{3}} &= x \\ 8 - x &= \sqrt{3}x \\ 8 &= (1+\sqrt{3})x \\ x &= \frac{8}{1+\sqrt{3}} = 4\sqrt{3} - 4 \end{aligned}$$

Now we can find $b = \sqrt{2}x = \sqrt{2}(4\sqrt{3} - 4) = \boxed{4\sqrt{6} - 4\sqrt{2}}$.

We can find $c = 2x = 2(4\sqrt{3} - 4) = \boxed{8\sqrt{3} - 8}$

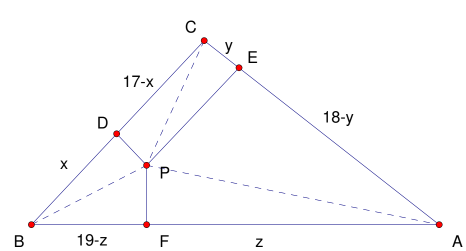
6. The trick to this one is to "'unfold"' the diagram as shown.



Since ABC is a 45-45-90, when we unfoldthe diagram above, we know that $\angle DAF = 90$, $\angle ADF = \angle AFD = 45$, and $\angle FBE = 90$, and $\angle BFE = \angle BEF = 45$. We can also see that $\angle CEF = 90$. So then $\angle CEB = \angle BPC = \angle CEF + \angle FEB = 90 + 45 = \boxed{135}$

7. $\boxed{24}$

8. Here be the diagram.



Set up the massive system of equations:

$AP^2 = PF^2 + z^2 = PE^2 + (18 - y)^2$ (1)

$BP^2 = PF^2 + (19 - z)^2 = PD^2 + x^2$ (2)

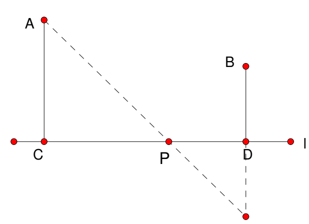
$CP^2 = PE^2 + y^2 = PD^2 + (17 - x)^2$ (3)

$x + y + z = 27$ (4)

$$
\begin{aligned}
(2) - (3): \quad PF^2 + (19 - z)^2 - PE^2 - y^2 &= x^2 - (17 - x)^2 \quad \text{subsitute } PF^2 \text{ and } PE^2 \text{ with (1)} \\
(19 - z)^2 - y^2 + (18 - y)^2 - z^2 &= 24x - 17^2 \\
19^2 - 38z + 18^2 - 36y &= 34x - 17^2 \\
34x + 36y + 38z &= 19^2 + 18^2 + 17^2 \\
34(x + y + z) + 2y + 4z &= 19^2 + 18^2 + 17^2 \quad \text{replace } x + y + z \text{ with 27} \\
2y + 4z &= 19^2 + 18^2 + 17^2 - 34(27) \\
2y + 4z &= 56 \\
y + 2z &= 28 \quad (5) \\
(5) - (4): z - x &= 1
\end{aligned}
$$

Then $BD + BF = 19 - z + x = 19 - (z - x) = 19 - 1 = \boxed{18}$

9. I know the problem doesn't have a question, but you should have been able to figure out that it was going to ask where to put P. What is the shortest distance between two points? A straight line! Let's manipulate the image so that we can draw a straight line.



Hooray! Just pop city B down below the line and connect the cities, and the intersection point is where P should be. We have similar triangles with a ratio of 1:2, since we know that the cities are 4 and 2 miles away from the freeway. So we know that the 8 mile freeway needs to be split such that $\boxed{PD = \frac{8}{3} \text{ and } CP = \frac{16}{3}}$

10. Time for some craaaazy algebra!

$2a^4 + 2b^4 + c^4 - 2a^2c^2 - 2b^2c^2 = 0$

$a^4 + b^4 + c^4 - 2a^2c^2 - 2b^2c^2 + 2a^2b^2 + a^4 + b^4 - 2a^2b^2 = 0$

$(c^2 - b^2 - a^2)^2 + (a^2 - b^2)^2 = 0.$

Since we have squared thing plus squared thing equals zero, the squared things can't be negative, so they both must be zero.

Since $c^2 - b^2 - a^2 = 0$, $c^2 = a^2 + b^2$, which means it is a right triangle.

Since $a^2 - b^2 = 0$, $a^2 = b^2 \Rightarrow a = b$, so it is an isoceles triangle.

So $\triangle ABC$ is $\boxed{\text{right isoceles}}$.