

Name: \_\_\_\_\_

Date: \_\_\_\_\_

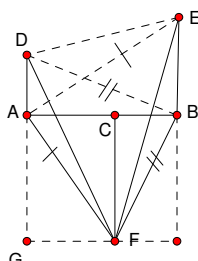
1. The following pairs of triangles are congruent:

$$\triangle AEF \cong \triangle ADF \quad \triangle ABF \cong \triangle ACF \quad \triangle ABG \cong \triangle ACG \quad \triangle BEF \cong \triangle CDF$$

$$\triangle ABD \cong \triangle ACE \quad \triangle EBC \cong \triangle DCB \quad \triangle BFG \cong \triangle CFG$$

So there are  $\boxed{7}$  pairs.

2. Draw the following lines.



Then  $\triangle AEB \cong \triangle AFC$ , which implies  $AE = AF$ . Also,  $\triangle AEB \cong \triangle AFB \cong \triangle AFC$ , so  $\angle EAF = \angle EAB + \angle BAF = 90^\circ$ , so  $\angle AFE = 45^\circ$ . So  $\angle BFE = 51 - 45 = 6^\circ$ , and  $\angle DFE = 51 - 2 \cdot 6 = \boxed{39^\circ}$

3.  $\boxed{3}$

4. Construct a line  $EF$  such that  $F$  is on  $AB$  and  $EF \parallel DA \parallel BC$ . Since  $AE$  and  $EB$  are angle bisectors of  $\angle DAB$  and  $\angle CBA$  respectively, and since they meet at a single point on  $CD$ , then  $\angle ADC = \angle BCD = 90^\circ$ . So we know then that  $EF \perp AB$ . So  $EF$  constructs two sets of congruent triangles,  $\triangle CEB \cong \triangle FEB$  and  $\triangle ADE \cong \triangle AFE$ . Then  $AF = DA = 4$  and  $BF = CB = 2$ , so  $AB = AF + FB = 4 + 2 = \boxed{6}$

5. We can split  $\triangle ACD$  into two 30-60-90 triangles. Then we can find  $BD = DC = \frac{\sqrt{3}}{3}$ . We can assume that  $\triangle MND$  is an equilateral triangle, and we can assume that  $BM = NC$ . Then we know that  $\angle BDM = \angle NDC = 30^\circ$ . So  $\triangle NCD$  is 30-60-90, and  $DC = \frac{\sqrt{3}}{3}$ . So  $NC = \frac{1}{3}$  and  $ND = 2NC = \frac{2}{3}$ , so the perimeter of  $MND = 3 \cdot \frac{2}{3} = \boxed{2}$