



Math Olympiad and Problem Solving Programs  
E230 - Advanced Math Competitions  
Problem Set 22.2 - Equation Traps

Name:

Date:

1. We can see that since  $x - 2$  is in the denominator,  $x \neq 2$ . Now when we solve, the equation, we see  $2x = 4 \Rightarrow x = 2$ . But we know that  $x \neq 2$ , so there is no solution.  $\boxed{\emptyset}$
2. Trap: square both sides, solve for  $x$ , get  $x = 11$ . But now when we plug  $x = 11$  back into the equation, we get  $3 = -3$ , which is not true. Furthermore, we know that the square root of something can never be negative, so we know that there is no solution.  $\boxed{\emptyset}$
3. We can immediately see that since  $2y - 7$  is in the denominator,  $y \neq \frac{7}{2}$ . So if  $y$  is any other value, the only way the two equations will be true is if the numerators equal zero, or  $x = 3$ . So we have one solution set of  $\boxed{\{(3, y) : y \neq \frac{7}{2}\}}$ .

Now let's consider when the denominator equals one, or  $y = 4$ . Then we have  $x - 3 = x - 3$ , which is true for all values of  $x$ . So we have a second solution set of  $\boxed{\{(x, 4) : x \in \mathbb{R}\}}$ .

(if you are not familiar with set notation,  $:$  means "such that,"  $\epsilon$  means "is an element of," and  $\mathbb{R}$  means "real numbers." So we read the second answer as: *the set  $(x, 4)$  such that  $x$  is an element of the real numbers*).

4. We can see that from the denominators,  $y \neq \frac{7}{2}$  and  $y \neq \frac{2}{7}$ . If  $y$  is any number but those two values, the only way the two equations will be the same is if they are both equal to zero. In other words,  $x = 3$ . So we write the solution set as  $\boxed{\{(3, y) : y \neq \frac{7}{2}, \frac{2}{7}\}}$ .

The other solution we can have is if both the denominators are the same, which occurs when  $y = 1$ . Then  $x - 3 = x - 3$  which is true for all  $x$ . So we have a second solution set  $\boxed{\{(x, 1) : x \in \mathbb{R}\}}$ .

5. When we solve things with absolute values, we split them into two equations. So solve both of the following:

$$(1) \sqrt{x} - \sqrt{2} < 1 \quad (2) \sqrt{x} - \sqrt{2} > -1$$

(1)

$$\begin{aligned} \sqrt{x} - \sqrt{2} &< 1 \\ \sqrt{x} &< 1 + \sqrt{2} \quad (\text{square both sides}) \\ x &< 3 + 2\sqrt{2} \end{aligned}$$

(2)

$$\begin{aligned} \sqrt{x} - \sqrt{2} &> -1 \\ \sqrt{x} &> -1 + \sqrt{2} \quad (\text{square both sides}) \\ x &> 3 - 2\sqrt{2} \end{aligned}$$

$$\boxed{3 - 2\sqrt{2} < x < 3 + 2\sqrt{2}}$$

6. Solve for  $|x|$ :  $2x - 1 = |x|$ . Split into two:  $2x - 1 = x$ ,  $-2x + 1 = x$ . Solve both:  $x = 1, \frac{1}{3}$ . Now we check by plugging both solutions into the original equation, and we see that the only solution is  $x = 1$

7. Since  $x < 0$ , let's consider where  $ax + 2 > 3x + b$  for positive  $x$  to make it easier on the brain. We want where  $ax > 3x$  and  $2 = b$ , which is when  $a > 3$ .

8. Let's find a pattern!

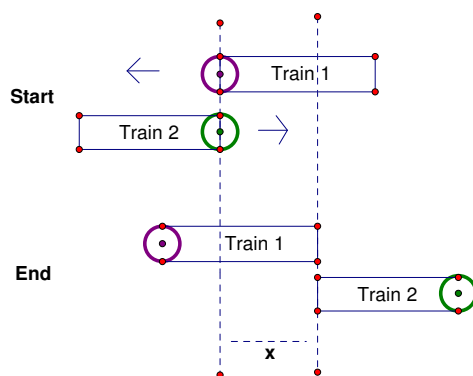
Numbers that leave a remainder of 1 when divided by 5: 6, 11, 16, 21, 26, 31, 36, ...,  $5n + 1, \dots$

Numbers that leave a remainder of 2 when divided by 7: 9, 16, 23, 30, 37, 42, 51, ...,  $7m + 2, \dots$

We can see from our lists that the first positive integer that has both properties is 16. We can see the next number that will be on both lists will be 51, because  $51 = 5n + 1$  for  $n = 10$ . Since the formulas  $5n + 1$  and  $7m + 2$  are linear, we can assume that the pattern will continue consistently. Since  $51 - 16 = 35$ , we can assume that a number satisfying both properties will occur every 35 numbers. (This makes sense since  $5 \times 7 = 35$ ). So numbers of this form are

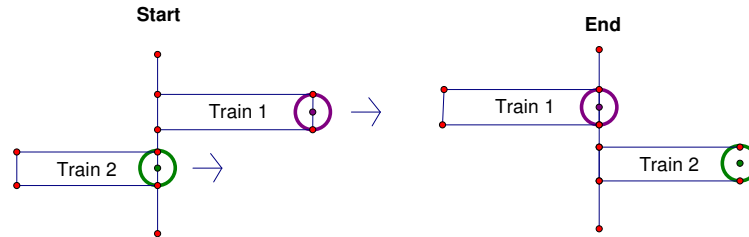
$$16 + 35k, k \in \mathbb{Z}$$

9. Let's draw some pictures.



Here is a picture of the trains passing each other. Let's consider the distance each train has traveled. To help us consider this idea, I have marked the front of each train with a purple or green circle dot thing. To consider how far the train has traveled is to consider how far the circle dot thing has traveled. Train 2, which is the faster, has traveled the distance  $x$  (the space between the vertical dotted lines) plus its own length. So  $d_2 = r_2t = L_2 + x$ . (here  $d = rt$  are the usual distance-rate-time variables, and  $L_i$  is the length of the  $i$ th train.) The distance that Train 1 has traveled is its own length minus  $x$ , or  $d_1 = r_1t = L_1 - x$ . If we add the two equations together, we get  $L_1 + L_2 = r_1t + r_2t$ . We are given that they pass each other in 8 seconds, or  $t = 8$ , and we are given the lengths of the two trains, so we can find that  $L_1 + L_2 = 350 + 450 = 800$ . So our first equation is  $800 = 8(r_1 + r_2) \Rightarrow 100 = r_1 + r_2$ .

Now let's consider the trains moving in the same direction.



Let's consider their distances again by observing how far the circle-dot-thing front has moved. In the first train, which I made slower since we decided it was slower in the first part, the distance it has traveled is just its rate times time, or  $d_1 = r_1 t$ . The second train has traveled 1) the length of train 1, 2) the distance traveled by train 1, 3) its own length, so  $d_2 = L_1 + d_1 + L_2 = L_1 + L_2 + r_1 t = r_2 t$ . Moving around, we get  $L_1 + L_2 = t(r_2 - r_1)$ . We are given  $t = 16$ , and  $L_1 + L_2 = 800$ , so  $800 = 16(r_2 - r_1) \Rightarrow 50 = r_2 - r_1$ .

So finally, we have the system of equations

$$\begin{aligned} 100 &= r_1 + r_2 \\ 50 &= r_2 - r_1 \end{aligned}$$

We solve for  $r_1$  since Train 1 is slower, and we get  $r_1 = \boxed{25 \text{ ft/s}}$

10. To make this problem easier, we are going to organize our work as follows. We will consider "noon today" to be the start of time, or  $t = 0$ . Then we will count in MINUTES (to make arithmetic easier) until "noon tomorrow," or  $t = 1440$  (which is  $24 \times 60$ ). So now we can take Dr. Li's riddle piece by piece, letting  $x$  be the time of now in minutes:

*If you add one-eighth of the time from noon until now...  $\frac{1}{8}(x - 0)$  (note that this implies the time is after noon)*

*...to one-quarter the time from now until noon tomorrow...  $\frac{1}{4}(1440 - x)$ .*

*...you get the time exactly.  $\frac{1}{8}(x - 0) + \frac{1}{4}(1440 - x) = x$ .*

Now we solve, and get  $x = 320$ , which is in minutes. And 320 minutes after 12:00pm is  $\boxed{5:20 \text{ p.m.}}$