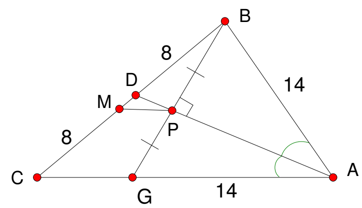
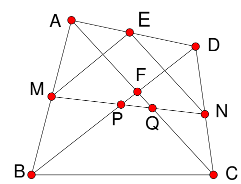
1. First we label the information in the diagram. Next, extend BP to intersect AC at G. Since AD is a bisector and $\overline{BP} \perp \overline{AD}$, then $AB = AG = 14$ and $BP = PG$. Then $\triangle BMP \sim \triangle BCG$. $\overline{CG} = 26 - 14 = 12$, and the ratio of the two triangles is 1:2, so $MP = \frac{1}{2}CG = \boxed{6}$



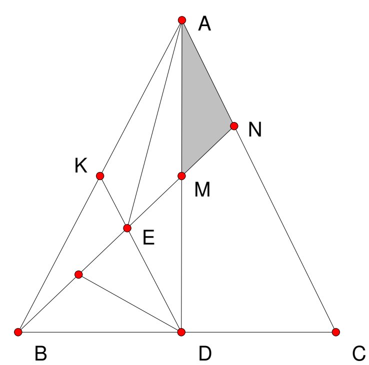
2. Draw the point E on AD such that $ME||BD$ and $NE||AC$.



Then $\angle FPQ = \angle FQP$ implies $\angle PME = \angle QNE$, which implies that $\triangle EMN$ is isosceles. Therefore, $EM = EN$. Then $AC = 2EM = 2EN = BD = \boxed{10}$

3. Since $[ABC] = 256$, $[A_1B_1C_1] = \frac{1}{4}256 = 64$. $[A_2B_2C_2] = \frac{1}{4}64 = 16$. $[A_3B_3C_3] = \frac{1}{4}16 = 4$. $[A_4B_4C_4] = \frac{1}{4}4 = 1$. $[A_5B_5C_5] = \frac{1}{4}1 = \boxed{\frac{1}{4}}$.

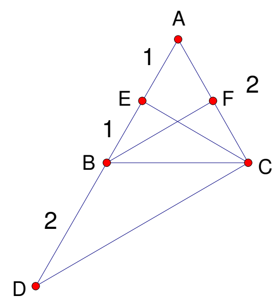
4. First, we draw the midpoint of AB, labeled K, and draw the midline DK. This midline splits $\triangle ABD$ in half, so $[KBD] = [AKD]$. We then split these two triangles into three smaller triangles each of equal area.



(diagram to scale)

Notice $[AMN] = [DME] = 40$, so we see that all three little triangles are the same size in $\triangle AKD$. So all 6 small triangles in $\triangle ABD$ have the same area 40. Half of the triangle is $40 \times 6 = 240$, so the area of the whole triangle is $\boxed{480}$.

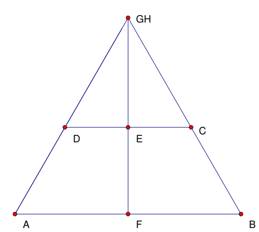
5. This is the diagram (to scale).



We see that $AE = EB = AF = AC = 1$ (arbitrary measurement), and $BF = CE$. Without loss of generality, let $\triangle ABC$ be equilateral. Then AEC is 30-60-90, and $EC = \sqrt{3}$. Also, $\triangle DEC$ is a right triangle, so $DC = \sqrt{3^2 + 3} = 2\sqrt{3}$. So $\frac{CD}{CE} = \frac{2\sqrt{3}}{\sqrt{3}} = \boxed{2}$

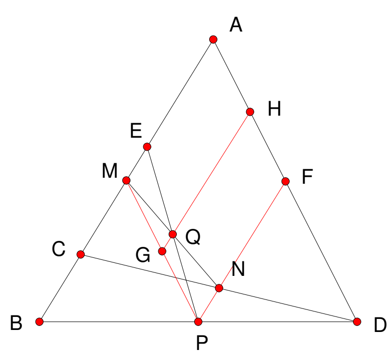
6. Without loss of generality, let $\triangle ABC$ be equilateral. Then D and E are midpoints, and DE is a midline. So AC is twice DE, so the ratio $\frac{AC}{DE} = \boxed{2}$

7. Without loss of generality, choose $ABCD$ to be an isosceles trapezoid. Then we have the following (to scale):



Clearly then $\frac{\angle AHF}{\angle BGF} = \boxed{1}$

8. Draw the line MP, its midpoint G, and the extension of GQ which meets line AD at H, and the extension of PN which meets AD at F.



(diagram to scale)

Notice $AM || HG || FP$. Also, GQ is the midline of $\triangle PEM$ and $\triangle MPN$. Therefore $ME = PN \Rightarrow AE = FN = \frac{1}{2}AC \Rightarrow \frac{AC}{AE} = 2$. Thus $\frac{EC}{AE} = \frac{AC-AE}{AE} = \frac{AC}{AE} - 1 = 2 - 1 = \boxed{1}$