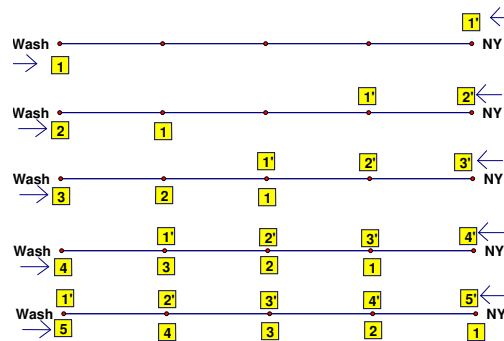


- 192 mph
- This problem is best examined by drawing a diagram. We draw a route from New York to Washington split into equal 4ths. We split it into 4ths because the trip is a 4 hour trip, and each train leaves on each hour. Now let's draw the trains in.



In my diagram, the trains with plain numbers leave Washington to New York, and the trains with apostrophes leave New York to Washington. Consider train 5. As it leaves Washington, it passes train 1' coming in from NY, and it will pass 2', 3', 4' it will meet 5' half way between the two cities. If you continue the pattern, train 5 will pass 6', 7', 8', and it will pass 9' as it pulls in to the NY station. So the train will pass 9 trains on its journey.

- 1:06 PM
- The total time elapsed 12 hours plus the number of hours lost. In other words, $12 + 11(\frac{1}{4}) =$
 $14\frac{3}{4}$ h
- Do you remember Dr. Li's Squares of Multiples of 5 Trick? If you have some multiple of 5 that end with 5 (ie 5, 15, 25, etc) squared, the procedure is as follows: for $(x5)^2$, where x are any digits:

- the last two digits are 25
- the digits before 25 are $x(x + 1)$.

For example, 75^2 , the last two digits are 25, and the digits before 25 are $7 \times 8 = 56$, so $75^2 = 5625$.

So let's examine the units digit of $x(x + 1)$. Let's consider each case for each units digit of x :

$$0 : 0(0 + 1) = 0. \quad 1 : 1(1 + 1) = 2. \quad 2 : 2(2 + 1) = 6. \quad 3 : 3(3 + 1) = 2. \quad 4 : 4(4 + 1) = 0. \\ 5 : 5(5 + 1) = 0. \quad 6 : 6(6 + 1) = 2. \quad 7 : 7(7 + 1) = 6. \quad 8(8 + 1) = 2. \quad 9(9 + 1) = 0.$$

So all the units digits of these expressions are either 0, 2, or 6. So if N is a square number, then $x = 0, 2, 6$. So there are 3 possibilities.

- 54 years

7. Let's try the equation with a few different values.

$$[6] + [1 - 6] = 6 + [-5] = 6 - 5 = 1.$$

$$[4.3] + [1 - 4.3] = 4 + [-3.3] = 4 - 4 = 0.$$

$$[0.4] + [1 - 0.4] = 0 + [0.6] = 0 + 0 = 0.$$

$$[-7.6] + [1 - (-7.6)] = -8 + [8.6] = -8 + 8 = 0.$$

So we can see that $[y] + [1 - y] = \boxed{1 \text{ for integers and } 0 \text{ for non integers}}$.

8. $\boxed{4 : 21 \frac{9}{11}}$

9. Let's find how long it takes for the hour and minute hand to overlap on an accurate clock. We will start at 12:00 when the hands begin overlapping. Then the minute hand will travel all the way around the clock and the hour hand will travel 30° to the 1:00 position. Now it is 1:00. The minute hand travels 6° per min, and the hour hand travels $.5^\circ$ per min. The hands will overlap in x minutes. The minute hand will travel $6x$ degrees and the hour hand will travel $30 + .5x$ degrees (we add 30 because its start position is 30° away from the minute hand's start position). Now we set them equal and solve: $6x = 30 + .5x \Rightarrow 5.5x = 30 \Rightarrow x = \frac{30}{5.5} = \frac{300}{55} = \frac{60}{11}$. So the time that has elapsed from 12:00 to overlap is $60 \text{ min} + \frac{60}{11} \text{ min} = \frac{720}{11} = 65 \frac{5}{11} \text{ min}$.

Now we use proportions:

$$\frac{65 \text{ min}}{65 \frac{5}{11} \text{ min}} = \frac{8 \text{ hours}}{n \text{ hours}}$$

where n is the accurate number of hours worked.

$$\text{So } n = \frac{8 \cdot 65 \frac{5}{11}}{65} = \frac{5760}{715} = \frac{1152}{143} = \boxed{8 \frac{8}{143} \text{ h} = 8 \text{ h } 3 \text{ min } 21.3 \text{ s}}.$$

10. It is very easy to get bogged down in a problem like this. You must resist the urge to find the daily ratios of tonic to water, or of water to mixture. These are difficult to find, and unnecessary! The essential clue is in the total number of ounces of water added during the drinking period.

On the first day, 1 oz. of water is added, and on the fifteenth day, 15 oz. of water are added. The total is, therefore, $1 + 2 + 3 + \dots + 15 = \frac{15 \cdot 16}{2} = \boxed{120 \text{ oz.}}$