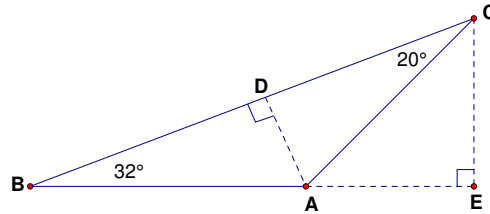


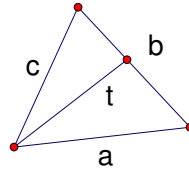
1. 6

2. The tricky part of this problem is drawing the diagram. Let's start with the information that tells us that  $\angle B = 32^\circ$  and  $\angle C = 20^\circ$ . Then we can see that  $\triangle ABC$  is obtuse with  $\angle A$  being the large measure angle. Now let's attempt to draw the triangle.



So we can easily find  $\angle EAD$ . Since  $\angle B = 32^\circ$  and  $\angle BDA$  is a right angle,  $\angle BAD = 58^\circ$ , and so  $\angle EAD = \boxed{122^\circ}$

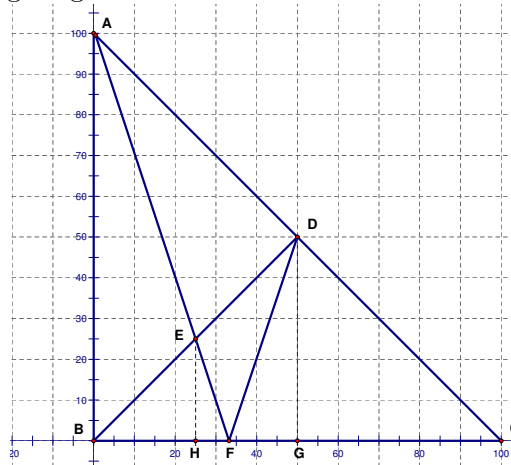
3. Angle Bisector Theorem (refer to the diagram below):  $t^2 = ac(1 - \frac{b^2}{(a+c)^2})$ .



So we use this formula for a 3-4-5 triangle.  $t^2 = 4 \cdot 3(1 - \frac{5^2}{(3+4)^2}) = 12(\frac{12}{49}) = \frac{288}{49}$ . So

$$t = \sqrt{\frac{288}{49}} = \boxed{\frac{12\sqrt{2}}{7}}$$

4. We can make this problem much easier by using an isosceles right triangle for the calculations. We can do this because if the ratio of areas is true for one triangle, it's true for all triangles. So we can pick a triangle that is easiest to work with. This is also a good time to use coordinate geometry. The following diagram is to scale.



Let's select a value for the side length of the triangle. I will choose 100. Then the  $[ABC] = .5 \cdot 100^2 = 5000$ .

Now we need to find the base and height of  $CDF$  in order to find its area. Notice that  $CD = 50\sqrt{2}$ . Also notice that  $\triangle DGC$  is a right isosceles triangle. So height  $DG$  is 50. To find the length of the base, we employ coordinate geometry. We know  $BD = 50\sqrt{2}$ , and  $E$  is the midpoint, so  $BE = 25\sqrt{2}$ . We also know that  $BHE$  is a isosceles right triangle, so  $BH = HE = 25$ . Thus, point  $E$ 's coordinates are  $(25, 25)$ . Consider the line  $AF$ . We want to find it's equation so we can find the point  $F$ . If we write  $AF$  in  $y = mx + b$  form, we know  $b = 100$ . We can find  $m$  because we have two points  $(0, 100)$  and  $(25, 25)$ , so we find that  $m = -3$ . So the equation of the line is  $y = -3x + 100$ . We want to know the coordinates of  $F$ , which is when  $y = 0 : 0 = -3x + 100 \Rightarrow x = \frac{100}{3}$ . So  $BF = \frac{100}{3}$ , and  $FC = 100 - \frac{100}{3} = \frac{200}{3}$ . So  $[FDC] = .5 \cdot 50 \cdot \frac{200}{3} = \frac{5000}{3}$ .

So the ratio  $[CDF]/[ABC] = \frac{5000}{3}/5000 = \boxed{\frac{1}{3}}$

5. Let  $r$  be the radius of the circle. Then the length of the diagonal of the square is  $2r$ . Then the area of the square is  $r^2$ .

If you have a circle inscribed in an equilateral triangle, the radius  $r$  of the circle and the side length  $s$  of the triangle are related as follows:  $r = s\frac{\sqrt{3}}{6}$ . So then  $s = 2\sqrt{3}r$ . Then the area of the triangle is  $\frac{1}{2} \cdot \frac{s}{2} \cdot (\frac{s}{2})\sqrt{3} = \frac{s^2\sqrt{3}}{8}$ , or  $\frac{(2\sqrt{3}r)^2\sqrt{3}}{8} = \frac{3\sqrt{3}}{2}r^2$ .

Finally, the ratio of  $[triangle] : [square] = \frac{3\sqrt{3}}{2}r^2 : r^2 = \frac{3\sqrt{3}}{2} : 1$  or  $\boxed{3\sqrt{3} : 2}$ .

6.  $\boxed{9\pi}$

7.  $\boxed{13}$

8.  $\boxed{45^\circ}$

9. Since  $\triangle ABD \sim \triangle ACD$ ,  $\frac{AB}{AD} = \frac{AC}{AB}$ , or  $AC = \frac{AB^2}{AD}$ . Since  $AF$  is an angle bisector,  $\frac{AB}{BE} = \frac{AD}{ED} \Rightarrow \frac{AB}{26} = \frac{AD}{10}$ . So  $AB = \frac{13}{5}AD$ . So  $AC = \frac{AB^2}{AD} = \frac{(\frac{13}{5}AD)^2}{AD} = \frac{169}{25}AD$ .

Now we solve for  $AD$  with the Pythagorean Theorem.

$$\begin{aligned} AD^2 + BD^2 &= AB^2 \\ AD^2 + 36^2 &= (\frac{13}{5}AD)^2 \\ AD^2 + 1296 &= \frac{169}{25}AD^2 \\ 1296 &= \frac{144}{25}AD^2 \\ 225 &= AD^2 \\ 15 &= AD \end{aligned}$$

So finally,  $AC = \frac{169}{25}AD = \frac{169}{25} \cdot 15 = \frac{2535}{25} = \boxed{\frac{507}{5}}$

10. The third side  $1 - 2x$  cannot equal or exceed  $3 + 4$ , and it cannot equal or be less than  $4 - 3$ . So  $1 < 1 - 2x < 7 \Rightarrow 0 < -2x < 6 \Rightarrow \boxed{0 > x > -3}$