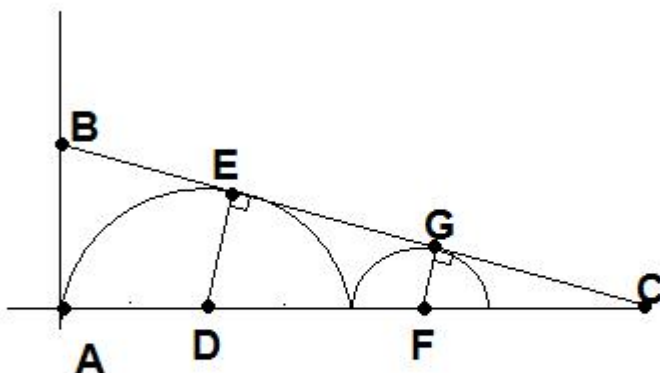


- Let s be the length of a side of the square. Consider an isosceles right triangle with vertices at the centers of the circle of radius 2 and two of the circles of radius 1. This triangle has legs of length 3, so its hypotenuse has length $3\sqrt{2}$. The length of a side of the square is 2 more than the length of this hypotenuse, so $s = 2 + 3\sqrt{2}$. Hence the area of the square is $s^2 = (2 + 3\sqrt{2})^2 = 22 + 12\sqrt{2}$. **[B]**
- [D]**
- Let E be the intersection of the diagonals of a rhombus $ABCD$ satisfying the conditions of the problem. Because these diagonals are perpendicular and bisect each other, $\triangle ABE$ is a right triangle with sides 5, 12, and 13, and area 30. Therefore the altitude drawn to side AB is $\frac{60}{13}$, which is the radius of the circle centered at E . **[C]**
- Let the side length of $\triangle ABC$ be s . Then the areas of $\triangle APB$, $\triangle BPC$, and $\triangle CPA$ are, respectively, $s/2$, s , and $3s/2$. The area of ABC is the sum of these, which is $3s$. The area of $\triangle ABC$ may also be expressed as $(\sqrt{3}/4)s^2$, and so $3s = (\sqrt{3}/4)s^2$. The unique positive solution for s is $4\sqrt{3}$. **[D]**
- Let D and F denote the centers of the circles. Let C and B be the points on the x and y-axes intersect the tangent line, respectively. Let E and G denote the points of tangency as shown.



We know that $AD = DE = 2$, $DF = 3$, and $FG = 1$. Let $FC = u$ and $AB = y$. Triangles FGC and DEC are similar, so

$$\frac{u}{1} = \frac{u+3}{2}$$

which yields $u = 3$. Hence $GC = \sqrt{8}$. Also, triangles BAC and FGC are similar, which yields

$$\frac{y}{1} = \frac{BA}{FG} = \frac{AC}{GC} = \frac{8}{\sqrt{8}} = \sqrt{8} = 2\sqrt{2}$$

D

6. Let a, b, c be the dimensions of the box. It is given that

$$140 = 4a + 4b + 4c \quad \text{and} \quad 21 = \sqrt{a^2 + b^2 + c^2},$$

hence,

$$35 = a + b + c \quad (1) \quad \text{and} \quad 442 = a^2 + b^2 + c^2 \quad (2).$$

Square both sides of (1) and combine with (2) to obtain

$$\begin{aligned} 1225 &= (a + b + c)^2 \\ &= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \\ &= 442 + 2ab + 2bc + 2ca. \end{aligned}$$

Thus the surface area of the box is $2ab + 2bc + 2ca = 1225 - 442 = 784$. B

7. First note that $FE = (AB + DC)/2$. Because trapezoids $ABEF$ and $FECD$ have the same height, the ratio of their areas is equal to the ratio of the averages of their parallel sides. Since

$$AB + \frac{AB + DC}{2} = \frac{3AB + DC}{2}$$

and

$$\frac{AB + DC}{2} + DC = \frac{AB + 3DC}{2},$$

we have

$$3AB + DC = 2(AB + 3DC) = 2AB + 6DC, \quad \text{and} \quad \frac{AB}{DC} = 5.$$

C