



Math Olympiad and Problem Solving Programs

E230 - Advanced Math Competitions

Grades 7 - 9

Lesson 17.1 - AMC 10 Algebra

Name:

Date:

1. Let's look at the sequence for a pattern:

$$a_1 = 2$$

$$a_2 = 2 + 2 = 4$$

$$a_3 = 4 + 4 = 8$$

$$a_4 = 8 + 6 = 14$$

$$a_5 = 14 + 8 = 22$$

⋮

Notice that we can get the following for a_5 :

$$a_5 = 22$$

$$= 14 + 8$$

$$= 8 + 6 + 4 + 4$$

$$= 8 + 6 + 4 + 2 + 2$$

And for a_4 :

$$a_4 = 14$$

$$= 8 + 6$$

$$= 6 + 4 + 4$$

$$= 6 + 4 + 2 + 2$$

The pattern shows that $a_n = 2 + [2 + 4 + 6 + \cdots + 2(n-1)] = 2 + 2[1 + 2 + 3 + \cdots + (n-1)]$.
We can plug in $n = 100$ to get:

$$a_{100} = 2 + 2[1 + 2 + 3 + \cdots + 99]$$

$$a_{100} = 2 + 2 \left[\frac{99(100)}{2} \right]$$

$$a_{100} = 2 + 9900 = 9902 \quad \boxed{\text{B}}$$

2. C

3. B

4. B

5. A

6. A

7. B

8. The difference between each a_{k-1} and a_k is a constant, call it m . Then we know that $a_7 = a_4 + 3m$, $a_{10} = a_4 + 6m$, giving us $3a_4 + 9m = 17$.

The second equation can similarly be changed into $11a_4 + 55m = 77$. We can now solve this system of equations to find $a_4 = \frac{11}{3}$, $m = \frac{2}{3}$. We can show that $a_k = a_4 + (k - 4)m$ similarly to how we changed all of our a_n into linear combinations of a_4, m . Now plugging in all the information we have:

$$13 = \frac{11}{3} + \frac{2}{3}(k - 4)$$

$$39 = 11 + 2(k - 4)$$

$$39 = 11 + 2k - 8$$

$$39 = 3 + 2k$$

$$36 = 2k$$

$$k = 18 \quad \text{B}$$

9. There are three different cases for which we can obtain 1 from our expression:

Case 1: $x^2 - x - 1 = 1$.

$$x^2 - x - 1 = 1$$

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$x = \boxed{-1, 2}$$

Case 2: $x + 2 = 0$. This obviously gives us $x = \boxed{-2}$.

Case 3: $x^2 - x - 1 = -1$ AND $(x + 2)$ is even. We first solve the quadratic and see which solutions give us $(x + 2)$ even:

$$x^2 - x - 1 = -1$$

$$x^2 - x = 0$$

$$x(x - 1) = 0$$

$$x = 0, 1$$

However, plugging them in we see that only $x = \boxed{0}$ gives us $(x + 2)$ even.

This is a total of 4 integers. C



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10. Since the roots of $x^2 + ax + 2b = 0$ are both real, this means that the discriminant, $a^2 - 8b$ is nonnegative. In other words $a^2 \geq 8b$. Similarly, the roots of $x^2 + 2bx + a = 0$ are both real so $4b^2 - 4b \geq 0$ or $b^2 \geq a$.

We can combine these inequalities if we first show $a^4 \geq 64b^2$ from squaring the first inequality and $64b^2 \geq 64a$ from multiplying the second inequality by 64.

This gives us $a^4 \geq 64b^2 \geq 64a$.

Using the fact that $a^4 \geq 64a$, we get that $a \geq 4$. If $a = 4$ then $b = 2$, satisfying all of the conditions. Then the minimum value for $a + b = 6$. E