1. Recall the sum formula for geometric series: $a\dfrac{1-r^n}{1-r}$. Here, $a = 3$ and $r = -2$. We need to find what term 768 is. The formula for the nth term of a geometric sequence is $a_n = ar^{n-1}$. Set them equal: $768 = 3(-2)^{n-1} \Rightarrow 256 = 2^8 = 2^{n-1} \Rightarrow 8 = n - 1 \Rightarrow n = 9$. So we calculate the sum: $a\dfrac{1-r^n}{1-r} = 3 \cdot \dfrac{1-(-2)^9}{1-(-2)} = 3 \cdot \dfrac{1+512}{3} = \boxed{513}$

2. $\boxed{400}$

3. Write the given information in the following form:

$$a_2 = ar = -2 \text{ and } a_5 = ar^4 = 16.$$

Now we solve for a and r. $a = \frac{-2}{r}$ from the first equation, so we substitute into the second and find $16 = (\frac{-2}{r})r^4 = -2r^3 \Rightarrow -8 = r^3 \Rightarrow r = -2$. Now we find a: $a = \frac{-2}{-2} = 1$. So the 14th term is $ar^{14-1} = 1(-2)^{13} = \boxed{\text{-8192}}$

4. $\boxed{\frac{48}{5} = 9.6}$

5. $\boxed{2184}$

6. $\boxed{67.5}$

7. Here is the general method for converting repeating decimals into fractions.

Set the decimal equal to x, so we have $x = 0.63\overline{36}$. Now multiply both sides by 10^n, where n is the number of repeating digits. We have 2 repeating digits here, so we multiply by $10^2 = 100$. So we get $100x = 63.\overline{36}$. Now we have a system of equations:

$$100x = 63.363636\ldots$$
$$x = 0.363636\ldots$$

When we subtract the second equation from the first, we get $99x = 63.36 - .63 = 62.73$. Divide both sides by 99 and we get $x = \frac{62.73}{99} = \frac{6273}{9900} = \boxed{\frac{697}{1100}}$

8. We will use the infinite sum formula to calculate the height of the bouncing. We have an initial value $a = 100$ and rate $r = \frac{3}{4}$. The sum of an infinite geometric series is $\frac{a}{1-r}$, so the sum of the distance the ball travels is $\dfrac{100}{1-\frac{3}{4}} = \dfrac{100}{\frac{1}{4}} = 400$.

A careless student would put down this answer and move on. But consider the action of a bouncing ball. It hits the ground, bounces up to its peak and falls back down again. So in order to calculate total distance traveled, we have to double the amount of distance, because the ball bounces up and then down. So the ball goes down 100, up 75, down 75, up 56.25, down 56.25, and so on. So we double what we found in our initial sum to 800, and then we must subtract 100 (because the ball only bounces down 100, it does not first travel up 100). So we end up with $\boxed{700}$

9. The formula for the nth term of a geometric sequence is $a_n = ar^{n-1}$. So we want to set $a_n = b$ and then solve for the first term, or a. So we write $b = ar^{n-1}$, and we divide both sides by r^{n-1} and we arrive at $\boxed{a = \frac{b}{r^{n-1}}}$.

10. $\boxed{\text{yes}}$